

Linear Systems

Solution	Can have no solution, one solution or infinitely many . The solution is the intersection
Solution set	The set of all possible solutions
Consistency	A system is consistent if there is at least one solution otherwise it is inconsistent
Equivalent	Linear systems are equivalent if they have the same solution set
Row operations	Replacement, interchange and scaling
Row equivalent	If there is a sequence of row operations between two linear systems then the systems are row equivalent . Systems that are row equivalent has the same solution set .
Existence	If a system has a solution (i.e. consistent)
Uniqueness	Is the solution unique
Homogenous	A system is homogenous if it can be written in the form $Ax = 0$
Trivial solution	If a system only has a the solution $x = 0$. A system with no free variable only have the trivial solution.
Non-trivial solution	A nonzero vector that satisfies $Ax = 0$. Has free variable.

Inverser of a Matrix

C is invertible if $CA = I^n$ and $AC = I^n$

If A is (2×2) then, $A^{-1} =$

$$(A^{-1})^{-1} = A$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A^T)^{-1} = (A^{-1})^T$$

Linear Transformations

Transformation/mapping	$T(x)$ from R^n to R^m
Image	For x in R^n the vector $T(x)$ in R^m is called the image
Range	The set of all images of the vectors in the domain of $T(x)$
Criterion for a transformation to be linear	1. $T(u + v) = T(u) + T(v)$ 2. $T(cu) = cT(u)$
Standard Matrix	The matrix A for a linear transformation T , that satisfies $T(x) = Ax$ for all x in R^n
Onto	A mapping T is said to be onto if each b in the codomain is the image of at least one x in the domain. Range = Codomain. Solution existence. Col A must match codomain.
One-to-one	If each b in the codomain is only the image at most one x in the domain . Solution Uniqueness. T is one-to-one if and only if the cols of A are linearly independent
Free variable?	If the system has a free variable, then the system is not one-to-one. I.e. the homogenous system only has the trivial solution
Pivot in every row?	Then T is onto

Linear Transformations (cont)

Pivot in every column?	Then T is one-to-one
To determine whether a vector c is in the range of a T . Solution: Let $T(x) = Ax$. Solve the matrix equation $Ax = c$. If the system is consistent , then c is in the range of T .	

The Invertible Matrix Theorem

The following statements are equivalent i.e. **either they are all true or all false**. Let A be a $(n \times n)$ matrix

A is an invertible matrix.

A is row equivalent to the $n \times n$ identity matrix

A has n pivot positions.

The equation $Ax = 0$ has only the trivial solution

The columns of A form a linearly independent set

The linear transformation $x \mapsto Ax$ is one-to-one.

The equation $Ax = b$ has at least one solution for each b in R^n

The columns of A span R^n

The linear transformation $x \mapsto Ax$ maps R^n onto R^n

There is an $n \times n$ matrix C such that \square
 $\square CA = I$.

There is an $n \times n$ matrix D such that \square
 $\square D = I$.

A^T is an invertible matrix.

The columns of A form a basis of R^n

Col $A = R^n$

Dim Col $A = n$

Rank $A = n$

Nul $A = \{0\}$

Dim Nul $A = 0$

The number 0 is not an eigenvalue of A

The determinant of A is not 0



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Elementary Matrices

Elementary Matrix Is obtained by performing a single elementary row operation on an **identity matrix**

Each elementary matrix **E** is **invertible**

A $n \times n$ matrix **A** is invertible if and only if **A** is row equivalent to I^n .

$$A = E^{-1}I^n \text{ and } A^{-1} = E I^n = I^n$$

Row reduce the augmented matrix $[A \ I]$ to $[I \ A^{-1}]$

NOTE If **A** is not row equivalent to **I** then **A** is not invertible

Linear Independence

A set of vectors are **linearly independent** if they cannot be created by any linear combinations of earlier vectors in the set.

If a set of vectors are **linear independent**, then the solution is **unique**

If the vector equation $c_1v_1 + c_2v_2 + \dots + c_p v_p = 0$ only has a **trivial solution** the set of vectors are **linearly independent**

Theorem: If a set contains more vectors than there are entries in each vector, then the set is **linearly dependent**

Theorem: If a set of vectors contain the zero vector, then the set is **linearly dependent**

Algebraic properties of a matrix

Algebraic properties of a matrix (cont)

$$(AB)^T = B^T A^T$$

$$\text{For any scalar } r, (rA)^T = rA^T$$

LU Factorization

Factorization of a matrix **A** is an equation that expresses **A** as a **product** of two or more matrices:

Synthesis: $BC = A$

Analysis: $A = BC$

Assumption: **A** is a $m \times n$ matrix that can be row reduced without **interchanges**

L: is a $m \times m$ unit lower triangular with 1's on the diagonal

U: is a $m \times n$ echelon form of **A**

U is equal to $E^*A = U$, why $A = E^{-1}U = LU$ where $L = E^{-1}$

See figure ** for how to find **L** and **U**

Find **x** by first solving $Ly = b$ and then solving $Ux = y$

Row Reduction and Echelon forms

Leading entry A leading entry refers to the leftmost **non-zero** entry in a row

Echelon form Row equivalent systems can be reduced into **several different** echelon forms

Reduced echelon form A system is only row equivalent to **one** REF

Forward phase Reducing an augmented matrix **A** into an **echelon form**

Backward phase Reducing an augmented matrix **A** into a **reduced echelon form**

Basic variables Variables in **pivot columns**.

Free variables Variables that are not in **pivot columns**. When a system has a free variable the system is **consistent** but not **unique**

Subspaces of R^n

A **subspace** of R^n is any set **H** in R^n that has three properties:

- The zero vector is in **H**

- For each **u** and **v** in **H**, the sum **u + v** is in **H**

- For each **u** in **H** and each scalar **c**, the vector **cu** is in **H**

Zero subspace is the set containing only the **zero vector** in R^n

Column space is the set of all linear combinations of the columns of **A**.

Null space (Nul A) is the set of all solutions of the equation $Ax = 0$

Basis for a subspace **H** is the set of **linearly independent** vectors that span **H**

In general, the **pivot columns** of **A** form a basis for **col A**

The number of vectors in any basis is **unique**. We call this number **dimension**

The **rank** of a matrix **A**, denoted by **rank A**, is the **dimension** of the **column space** of **A**

Determine whether **b** is in the **col A**.

Solution: **b** is only in **col A** if the equation $Ax = b$ has a solution

Matrix and vector sum	$A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$
Matrix, vector and scalar	$A(c\mathbf{u}) = c(A\mathbf{u})$
Associative law	$A(BC) = (AB)C$
Left distributive law	$A(B + C) = AB + AC$
Right distributive law	$(B + C)A = BA + CA$
Scalar multiplication	$r(AB) = (rA)B = A(rB)$
Identity matrix multi	$I^m A = A = A I^n$
Commute	If $AB = BA$ then we say that A and B commute with each others
	$(A^T)^T = A$
	$(A + B)^T = A^T + B^T$



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Published 27th April, 2026.

Last updated 11th June, 2022.

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Algebraic properties of a vector

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$$

$$\mathbf{u} + (-\mathbf{u}) = -\mathbf{u} + \mathbf{u}$$

$$c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$$

$$c(d\mathbf{u}) = (cd)\mathbf{u}$$



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