Cheatography

Sequences and Series Cheat Sheet

by ebabor via cheatography.com/69500/cs/17544/

General Rules

Telescoping and Geometric series are the only types of series that you can estimate sums from. So, you must use these test's properties to estimate these sums

If the question is asking for absolute convergence or conditional convergence. You will need to use the Ratio Test, Root Test, or the definition of Absolute/Conditional Convergence

Must show **ALL** work to receive full credit for questions. Study the process to solve the problems, don't just guess through the review

Tests

Test for Divergence(TFD)	Inconclusive	You absolutely cannot determine if a series is convergent from this test.
	Diverges	If limit of series $\neq 0$ or ∞
Integral Test	Converges	If integral of series $<\infty$
	Diverges	If integral of series $=\infty$
Ratio Test	Converges/ Converges Absolutely	If limit of $0\leq\lvert(a_{k+1})/(a_k)\rvert<1$
	Diverges	If limit >1
	Inconclusive	If limit $=1$
Root Test	Converges/ Converges Absolutely	If $0\leq$limit of the k^{th} root of $\lvert a_k\rvert<1$
	Diverges	If limit of the k^{th} root of $\lvert a_k\rvert>1$
	Inconclusive	If limit of k^{th} root of $\lvert a_k\rvert=1$
Direct Comparison Test(CT)	Converges	If $\sum b_k$ converges AND b_k is the larger of the two functions

Tests (cont)

	Diverges	If $\sum b_k$ diverges AND b_k is smaller of two functions
Limit Comparison Test(LCT)	Converges	If b_k converges AND limit of $0<(a_k)/(b_k)<\infty$
	Diverges	If b_k AND limit of $0<(a_k)/(b_k)<\infty$
Alternating Series Test(AST)	Converges	If all 3 conditions for AST are met
	Diverges	If limit condition fails, $\sum a_k$ is immediately divergent by TFD

Properties of Special Series

Geometric Series	Converges	If Absolute Value of $r<1$. Converges at S=(first term)/(1-r)
	Diverges	If Absolute Value of $r\geq1$
P-Series	Converges	If $p>1$
	Diverges	If $p\leq1$
Telescoping	Converges	Value that the limit of the remaining terms approach
	Diverges	Almost never. On the test it will converge
Definition of Convergence	Absolute Convergence	If and only if $\sum\lvert a_k\rvert$ is convergent
	Conditional Convergence	If and only if $\sum a_k$ is convergent, but $\sum\lvert a_k\rvert$ is divergent

When to Use Tests

Properties	If you can identify the series as a geometric, p, or telescoping series, then use their respective properties. If the given series looks *close* to one of these series **see if you can use algebra to rearrange it into one of them**
Test for Divergence(TFD)	Should at least eyeball this test first to see if the limit of the series does not approach 0. If series does not approach 0, then **$\sum a_k$ divergent by TFD**
Comparison Tests(CT and LCT)	**ONLY POSITIVE TERMS!** If you can tell if the series has negative terms,($(-1)^k$ or sin/cos), do not use this test. If series has is rational and has a root in the denominator, compare with a p-series. $\lvert a_k\rvert$ gives use absolute convergence
Alternating Series Test(AST)	Series with $(-1)^k$ can be testes with AST
Integral Test	**ONLY POSITIVE TERMS!** If you can look at the function and easily take the integral, it is probably good to use this test
Ratio Test	If series contains: k!, or powers *and* exponentials, **almost guaranteed to use ratio test**

When to Use Tests (cont)

Root Test	If the entire series can be written to the k^{th} power, you can use the root test

Integral Test

Conditions:
1. f(x) is positive on its interval
2. f(x) is continuous on its interval
3. f(x) decreasing as x->∞ (derivative is negative)

* Must change a_k to a function in order to take derivative
* Integral starts off from k to ∞, so you must change the integral to k to t with limit as t->∞
* Answer you get is not where the $\sum a_k$ converges

Alternating Series Test

Conditions:
1. $b_k > 0$
2. $b_k \geq b_{k+1}$
3. limit of $b_k = 0$

* If $\sum b_k$ fits all three conditions, $\sum a_k$ convergent by AST
* If 3rd condition fails, $\sum a_k$ is divergent by TFD
* If series contains $(-m)^k$, pull $(-1)^k$ out and keep m^k in b_k

Integral Remainder

Known n^{th} Value	Solving for S_n (sum of the series approximation)	Plug n into the series
	Solving for R_n (approximation of the remainder)	Solve integral from n to ∞
Known Error Bound	Set the integral of f(x) from n to ∞ less than error bound. Once solved, answer will be given in terms of n<#. Must round up the number since series only use integers and if you rounded down, the value of the integral would be larger than the error bound	

Alternating Series Remainder

Known Error Bound	Set error bound less than b_{n+1}. Solve for a #>n, round up n to next highest whole number
	If the inequality is very difficult to solve, the use of a table, shown below, is acceptable. When the middle column, b_{n+1}, is less than third column, error bound, then that value is you final answer for n. Since the original variable in the equation is k, and k=n+1, then the value of the final term you can stop on to reach your error bound will be k

n^{th} term	b_{n+1}	Error Bound