

Binomial & Bernoulli Distributions

Introduction:

Given an experiment with 2 possible outcomes (Success & Failure ie. 1 and 0 ie. Binary) ran 5 times.

Sample space Ω becomes a combination of the 5 results ie. $\Omega = \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}$.

With the information that event A is when any one experiment is a success we are able to deduce that set $A = \{(0,0,0,0,1), (0,0,0,1,0), (0,0,1,0,0), (0,1,0,0,0), (1,0,0,0,0)\}$

Equate the probability of success happening (not necessarily in A, only in general as a result of the experiment being ran any 1 time) is given by p, and consequently (as there are only 2 possibilities) failure is given by 1-p. Remember p is the probability of a single result occurring that is considered a success. In the case we are flipping a coin and heads is a success $p = P(H) = 1/2$, in the case of a dice where 5 is considered a success $p = P(5) = 1/6$

We can obviously see that $P(A) = 5(1-p)^4p$ or in plain english - "There are 5 possible combinations of A occurring, contained within each 4 failures $(1-p)^4$ and 1 success p"

Bernoulli Distribution...

- Used to model experiments with binary outcomes (success or failure)
- Definition: Discrete rv X has a Bernoulli distribution with parameter p (p is the probability of success occurring in any given experiment) where $0 < p < 1$ if its probability mass function (pmf) is given by...

$$p_X(1) = P(X=1) = p$$

and

$$p_X(0) = P(X=0) = 1-p$$

ie. Probability of success occurring is p and probability of failure is 1-p

Binomial Distribution:

A discrete rv X has a binomial distribution with parameters n and p where $n = 1, 2, \dots$ and $0 < p < 1$, if the pmf is given by... $P_X(k) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

Denoted by $\text{Bin}(n, p)$

Expectation of binomial distribution $\text{Bin}(n, p)$ is $E(X) = np$

Variance is $\text{Var}(X) = np(1-p)$

Hypergeometric Random Variable

Introduction:

When an experiment consists of...

1. Drawing n random elements WITHOUT REPLACEMENT from a set of N elements.
2. s elements of N are special.
3. N - s are regular

Result: Our Hypergeometric rv X is the number of special elements in our random draw of n ie $s_{\cap n}$ (sortof)

The Geometric Distribution

Introduction:

1. Observations have 2 possible values Success and Failures $\{\{n\}\}^2$.

Probability of success (p) is the same for each observation.

Observations are all independent.

We are looking for the number of trials required to find the first success.

This is an example of an infinite experiment...

A discrete rv X has a geometric distribution with parameter p, where $0 < p < 1$, if its pmf is given by

$$p_X(k) = P(X=k) = (1-p)^{k-1}p$$

for $k=1, 2, \dots$

This distribution is denoted by $\text{Geo}(p)$ $E(X) = \sum_{k=1}^{\infty} kp(1-p)^{k-1} = 1/p$

$$\text{Var}(X) = (1-p)/p^2$$

Memoryless Property:

$$P(X > n+k | X > k) = P(X > n)$$

Poisson Distribution

Introduction:

Used when we are interested in the number of X per the number of Y. eg. The number of cars on a road per hour. This is phrase as counts per unit interval.

When count is relatively low it can be modelled as a Poisson Distribution.

Assumptions: $\{\{n\}\}^1$. Homogeneity: The rate λ (the rate at which the event occurs) is constant over time/space. This implies that at any unit interval $E(X) = \lambda$

2. Independence: Number of events in disjoint intervals are independent of one another.

Please refer to Definitions and Other Formulae sheet for more information.

Property of Poisson:

If we sum 2 Poisson rv X and Y, our result is also Poisson.

$\{\{n\}\}^x \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ then $X+Y \sim \text{Poisson}(\lambda+\mu)$

