DefinitIons

Finite Automaton(FA)

5-tuple(Q,Σ,δ,q,F) where
i)Q = states
ii)Σ = input alphabet
iii)δ:QxΣ -> Q = transitions
iv)qϵQ = start state
v)F = accept states

Regular Language(RL)

Is recognized by some FA and is closed under the operations:
i)∪
ii)∘
iii)*
iv)∩
v)Complementation

Deterministic Finite Automaton(DFA)

FA where there is only one state that can be transitioned to from the current state and current input symbol.

Nondeterministic Finite Automaton(NFA)

FA where there is one or more states that can be transitioned to from the current state and current input symbol.

Power Set

The set of all subsets of a language.
Has size $2^{|A|}$.

Regular Expression(RE)

Is one of the following:
i)a such that aϵΣ
ii)ε
iii)∅
iv)S∪T where S,T are RE
v)S∘T where S,T are RE
vi)S* where S is a RE

DefinitIons (cont)

Generalized NFA(GNFA)

5-tuple(Q,Σ,δ,q,a) where
i)Q =states
ii)Σ = input alphabet
iii)δ:(Q-{a})x(Q-{q}) -> R = transitions
iv)qϵQ = start state
v)aϵQ = accept state

Also must meet the following conditions
i)Start state has transitions arrows going out, but none coming into itself
ii)There is only one accept state with transition arrows coming in, but none going out from itself
iii)Except for the start and accept states, only one arrow goes from every state to every other state, including an arrow to itself

CFL Definitions

Context Free Languages(CFL)

Languages described by context free grammars and pushdown automata. They include all RL.
Any language that can be generated by a CFG.

Context Free Grammar(CFG)

4-tuple(V,Σ,R,S), where:
i)V = set of variables
ii)Σ = set of terminals, disjoint from V
iii)R = set of rules, with each rule being a variable and a string of variables and terminals
iv)SϵV = start variable

Ambiguity

A string is ambiguous if there is more than one way to generate the string. If a CFG has an ambiguous string, then the grammar is ambiguous.
If an ambiguous CFG generates a CFL that can only be generated by ambiguous grammars, then it is **inherently ambiguous**.

Leftmost Derivation

At each step of string derivation, the leftmost variable is replaced.

CFL Definitions (cont)

Chomsky Normal Form(CNF)

Every rule is of the form A->BC or A->a, where a is a terminal and A,B,C are variables. We allow S->e, where S is the start variable.

Pushdown Automaton(PDA)

6-tuple(Q,Σ,Γ,δ,q,F), where
i)Q = states
ii),Σ, = input alphabet
iii)Γ = stack alphabet
iv)δ:QxΣxΓ -> P(QxΓ) = transition function
v)qϵQ = start state
vi)FϵQ = accept states

By definition, they are nondeterministic.

Theorems, Lemmas, Corallaries for RL

Every NFA has an equivalent DFA

A language is regular iff some NFA recognizes it

If a language is described by a RE, then it is regular

If a language is regular, it is described by a RE

A language is regular iff some RE describes it

Pigeonhole Principle: If we stuff n items into less than n holes, at least one hole must have more than one item in it

There is an algorithm that can determine if two FA are equivalent

Pumping Lemma for RL

Let A be a RL. Then there exists a number p(pumping length of A), such that any string from the language A having length at least p can be broken into three pieces, s=xyz such that:

i)∀ i≥0, xy^iz is an element of A
ii)|y|>0
iii)|xy|≤p

Theorem, Lemmas, Corallaries for CFL

Any CFL can be generated by a CFG in Chomsky Normal Form.

A language is context free(CFL) iff some PDA recognizes it.

If a language is context free(CFL), then some PDA recognizes it.

If a PDA recognizes some language, then the language is contex free(CFL).

Every regular language is context free.

Pumping Lemma for CFL

If A is a CFL, then there is a number p(pumping length of A) where, if s is any string in A with length at least p, then s may be divided into 5 pieces, s = uvxyz where:

> i)∀ $i \geq 0$, uv^ixy^iz is an element of A
> ii)$|vy|>0$
> iii)$|vxy| \leq p$

Contradictions by condition:

> ii)At least one of v and y cannot be the empty string
> iii)Can be used in showing some languages are not CFL