Differentiation

gradient of a line	m = rise/run = (y2-y1)/(x2 -x1)
as lim approaches 0	m = (lim h→0) f(x + h) - f(x)/h
first derivative	f'(x) = df/dx
second derivative	f''(x) = d^2f/dx^2
third derivative	f'''(x)=d^3f/dx^3
d/dx x^n = nx^{n-1}	d/dx ln(x) - 1/x
d/dx e^x = e^x	d/dx sin(x) = cos(x)
d/dx cos(x) = -sin(x)	
product rule	y= uv
	y'uv' + vu'
chain rule	y = y(u(x))
	dy/dx = dy/du . du/dx
quotient rule	y = u/v
	y' = u'v - uv'/v^2

rewrite gradient of line: m= f(x+h) - f(x)/h

scalar product rule d/dx (ku(x)) = ku'(x) where k is a scalar
derivative of a sum: d/dx. (u(x)+v(x)) = u'(x)+v'(x)

Vectors

sin(θ) = opposite/hypotenuse
cos(θ) = adjacent/hypotenuse
tan(θ0= opposite/adjacent
$a^2+b^2=c^2$

Matrices

C = A+B	addition/subtraction
$B = \square \square A$	k is scalar, A is $m\ .\ n$ matrix
$C = \square \square B$	if A = $m\ .\ n$, B = $n\ .\ k$

Trig Functions

y = a sin(bx + c) + d	y = a cos(bx + c) + d
exponential function	y = e^x
domain	values x can assume
range	values y can assume

amplitude = a
period = 2π/b
horizontal shift = - c/b
vertical shift = d

sin(x) starts at 0, cos(x) starts at one

Expon - e = eulers's constant.

domain/range : _ (> or <) _

Logarithmic Differentiation

ln(ab) = ln(a) + ln(b)
ln(a/b) = ln(a) - ln(b)
ln(a^b) = b x ln(a)
ln(e) = 1
$e^{ln(x)}$ = x

Area Between Curves

∫f(x)dx - ∫g(x)dx	f(x) = upper function g(x0 = lower function
Volume of Revolution	V = π ∫ y^2 dx
Integrating Ration Functions	f'(x) = x/x^2-1

Integrals

∫sin(x)dx	-cos(x) + C
∫cos(x)dx	sin(x) + C
∫e^x dx	e^x + C
∫1/x dx	ln(x) + C
∫x^n dx	x^{n+1}/n+1 + C
∫ln(x) dx	xln(x) - x + C
scalar rule	∫ ku(x) dx = k∫u(x) dx
integral of a sum	∫(u(x) + v(x))dx = ∫u(x)dx +∫v(x)dx
derivative of intergral	d/dx∫u(x)d x= u(x)
integral of derivative	∫u'(x)dx =u (x) + C

Integrals of Common Functions

∫sin(nx) dx	-1/n cos(nx) +C
∫cos(nx) dx	1/n sin(nx) = C
∫e^{nx} dx	1/n e^{nx} + C
∫ln(nx)dx	1/n ln(nx) + C

Integration by Substitution

∫y(u(x))u'(x)dx	∫y(u)du

Integration by Parts

∫uv' dx = uv- ∫u'v dx

∫x^ndx = x^{n+1}/n+1 + C only applies when n does NOT equal -1

when n= -1, ∫1/x dx applies

Indefinite Integral: no numbers at top of bottom.

Definite Integral: solve for a number that represents the areas under the curve from x=a to x=b
no integration constant in this situation

rules

product rule: x multiplied together in different forms eg. $y = e^2e^x$

chain rule:
inner function u(x)
outer function: y(u)

looking for function within a function eg. y=ln(sin(x)).
let u equal the inner function

quotient: x in both the numerator and denominator eg. $y = e^xx^2$

remember $1/a^n = a^{-n}$

Functions & Algebraic Structure

y-intercept: where crosses y	solve for y when x = 0
roots: where crosses x	solve for x when y = 0
linear functions	y = mx + c
quadratic functions	$y = ax^2 + bx + c$
turning point	x = -b/2 . a
roots of quadratic	use quadratic formula
2π = 360°	radians = degrees . π/180

Function – can have only one output, y, or each unique input, x.
Relation - can have more than one output, y, for each unique input, x.

may be be more than one root for a function. roots can also be called x-intercepts and zeros

linear: mx= gradient/slope C= y-intercept

quadratic: pos a = 'happy face', neg a = 'sad face'

Explicit/Implicit

Explicit: dependent variable is written explicitly in terms of the independent.
eg. y = 3x + 5

Implicit: dependent variable is not isolated to one side of equation
eg. 3x + 5 - y = 0

Explicit differentiation: when starting with implicit from that is rearrangeable, rearrange then do.

Implicit differentiation: relies on the chain rule. No rearranging required

Differential Equations

First Order Separable

f(x) dx = g(y) dy	put all x to one side and y to other

Power & Log Rules

$a^b . a^c = a^{b+c}$

$a^b/a^c=a^{b-c}$

$ln(a^b)= bln(a)$

ln(e) = 1

$e^{ln(x)} = x$

Decay

dN/dt = -λN	N = amount of substance, t = time and λ is decay constant

Newton's Law of Cooling

dT/dt = -k(T-Ta)	T = Temp of object, Ta is ambient temp, t is time a k is heat transfer constant

**Motion Problems*

v = ds/dt	s = position, v = velocity, a = acceleration, t= time

Differential Equations (cont)

a = dv/dt

A differential equation is just a mathematical equation that involves derivatives.

can have more than one solution