

Differentiation

gradient of a line	$m = \text{rise/run} = (y_2 - y_1)/(x_2 - x_1)$
as $h \rightarrow 0$ approaches 0	$m = (\lim_{h \rightarrow 0} f(x+h) - f(x))/h$
first derivative	$f'(x) = df/dx$
second derivative	$f''(x) = d^2f/dx^2$
third derivative	$f'''(x) = d^3f/dx^3$
$d/dx x^n = nx^{n-1}$	$d/dx \ln(x) = 1/x$
$d/dx e^x = e^x$	$d/dx \sin(x) = \cos(x)$
$d/dx \cos(x) = -\sin(x)$	

product rule	$y = uv$ $y' = uv' + vu'$
chain rule	$y = y(u(x))$ $dy/dx = dy/du \cdot du/dx$
quotient rule	$y = u/v$ $y' = u'v - uv'/v^2$

rewrite gradient of line: $m = f(x+h) - f(x)/h$

scalar product rule $d/dx (ku(x)) = ku'(x)$
where k is a scalar
derivative of a sum: $d/dx (u(x)+v(x)) = u'(x)+v'(x)$

Vectors

$\sin(\theta) = \text{opposite/hypotenuse}$
$\cos(\theta) = \text{adjacent/hypotenuse}$
$\tan(\theta) = \text{opposite/adjacent}$
$a^2 + b^2 = c^2$

Matrices

$C = A+B$	addition/subtraction
$B = kA$	k is scalar, A is $m \cdot n$ matrix
$C = AB$	if $A = m \cdot n$, $B = n \cdot k$

Trig Functions

$y = a \sin(bx + c) + d$	$y = a \cos(bx + c) + d$
exponential function	$y = e^x$
domain	values x can assume
range	values y can assume

amplitude = a
period = $2\pi/b$
horizontal shift = $-c/b$
vertical shift = d

$\sin(x)$ starts at 0, $\cos(x)$ starts at one

Expon - e = eulers's constant.

domain/range : $_ (> \text{ or } <) _$

Logarithmic Differentiation

$\ln(ab) = \ln(a) + \ln(b)$
$\ln(a/b) = \ln(a) - \ln(b)$
$\ln(a^b) = b \times \ln(a)$
$\ln(e) = 1$
$e^{\ln(x)} = x$

Area Between Curves

$\int f(x)dx - \int g(x)dx$	$f(x) = \text{upper function}$ $g(x) = \text{lower function}$
Volume of Revolution	$V = \pi \int y^2 dx$
Integrating Ration Functions	$f'(x) = x/x^2 - 1$

Integrals

$\int \sin(x)dx$	$-\cos(x) + C$
$\int \cos(x)dx$	$\sin(x) + C$
$\int e^x dx$	$e^x + C$
$\int 1/x dx$	$\ln(x) + C$
$\int x^n dx$	$x^{n+1}/n+1 + C$
$\int \ln(x) dx$	$x \ln(x) - x + C$
scalar rule	$\int ku(x) dx = k \int u(x) dx$
integral of a sum	$\int (u(x) + v(x))dx = \int u(x)dx + \int v(x)dx$
derivative of intergral	$d/dx \int u(x) dx = u(x)$
integral of derivative	$\int u'(x)dx = u(x) + C$

Integrals of Common Functions

$\int \sin(nx) dx$	$-1/n \cos(nx) + C$
$\int \cos(nx) dx$	$1/n \sin(nx) + C$
$\int e^{nx} dx$	$1/n e^{nx} + C$
$\int \ln(nx)dx$	$1/n \ln(nx) + C$

Integration by Substitution

$$\int y(u(x))u'(x)dx = \int y(u)du$$

Integration by Parts

$$\int uv' dx = uv - \int u'v dx$$

$\int x^n dx = x^{n+1}/n+1 + C$ only applies when n does NOT equal -1

when $n = -1$, $\int 1/x dx$ applies

Indefinite Integral: no numbers at top of bottom.

Definite Integral: solve for a number that represents the areas under the curve from $x=a$ to $x=b$
no integration constant in this situation



rules

product rule: x multiplied together in different forms eg. $y = e^2e^x$

chain rule:

inner function $u(x)$
outer function: $y(u)$

looking for function within a function eg.

$y = \ln(\sin(x))$.

let u equal the inner function

quotient: x in both the numerator and denominator eg. $y = e^x x^2$

remember $1/a^n = a^{-n}$

Functions & Algebraic Structure

y-intercept: where solve for y when x crosses y = 0

roots: where crosses solve for x when y x = 0

linear functions $y = mx + c$

quadratic functions $y = ax^2 + bx + c$

turning point $x = -b/2 \cdot a$

roots of quadratic use quadratic formula

$2\pi = 360^\circ$ radians = degrees . $\pi/180$

Function – can have only one output, y, or each unique input, x.

Relation - can have more than one output, y, for each unique input, x.

may be be more than one root for a function. roots can also be called x-intercepts and zeros

linear: $mx =$ gradient/slope $C =$ y-intercept

quadratic: pos a = 'happy face', neg a = 'sad face'

Explicit/Implicit

Explicit: dependent variable is written explicitly in terms of the independent.

eg. $y = 3x + 5$

Implicit: dependent variable is not isolated to one side of equation

eg. $3x + 5 - y = 0$

Explicit differentiation: when starting with implicit from that is rearrangeable, rearrange then do.

Implicit differentiation: relies on the chain rule. No rearranging required

Differential Equations

First Order Separable

$f(x)$ put all x to one side and y to other
 $dx =$

$g(y)$

dy

Power & Log Rules

$a^b \cdot a^c = a^{b+c}$

$a^b/a^c = a^{b-c}$

$\ln(a^b) = b \ln(a)$

$\ln(e) = 1$

$e^{\ln(x)} = x$

Decay

dN/dt $N =$ amount of substance, $t =$ time
 $= -\lambda N$ and λ is decay constant

Newton's Law of Cooling

dT/dt $T =$ Temp of object, T_a is ambient
 $= -$ temp, t is time k is heat transfer constant
 $k(T - T_a)$

*Motion Problems

$v =$ $s =$ position, $v =$ velocity, $a =$

ds/dt acceleration, $t =$ time

Differential Equations (cont)

$a = dv/dt$

A differential equation is just a mathematical equation that involves derivatives.

can have more than one solution

