Circular Functions Definitions

Name	Right-Triangle Definition	Domain	Range
Sine Function	sin(θ)=o/h	(-∞,∞)	[-1,1]
Cosine Function	cos(θ)=a/h	(-∞,∞)	[-1,1]
Tangent Function	tan(θ)=o/a	{θ|θ≠π/2±πn}	(-∞,∞)
Cosecant Function	csc(θ)=h/o	{θ|θ≠±πn}	(-∞,-1]∪[1,∞)
Secant Function	sec(θ)=h/a	{θ|θ≠π/2±πn}	(-∞,-1]∪[1,∞)
Cotangent Function	cot(θ)=a/o	{θ|θ≠±πn}	(-∞,∞)
Inverse Sine Function	arcsin(o/h)=θ	[-1,1]	[-π/2,π/2]
Inverse Cosine Function	arccos(a/h)=θ	[-1,1]	[0,π]
Inverse Tangent Function	arctan(o/a)=θ	(-∞,∞)	(-1,1)
Inverse Cosecant Function	arccsc(h/o)=θ	(-∞,-1)∪(1,∞)	[-π/2,0)∪(0,π/2]
Inverse Secant Function	arcsec(h/a)=θ	(-∞,-1)∪(1,∞)	[0,π/2)∪(π/2,π]
Inverse Cotangent Function	arccot(a/o)=θ	(-∞,∞)	(0,1)
Circular Euler Relation	$e^{\pm i\theta}$=cos(θ)±isin(θ)		
De Moivre's Theorem	$e^{in\theta}$=$(\cos(\theta)+i\sin(\theta))^n$=cos(nθ)+isin(nθ)		

$n \in \mathbb{N}_1$ = {1,2,3,4,5,...}

Hyperbolic Functions Definitions

Name	Exponential Definition	Domain	Range
Hyperbolic Sine Function	sinh(θ)=$(e^{\theta}-e^{-\theta})/2$	(-∞,∞)	(-∞,∞)
Hyperbolic Cosine Function	cosh(θ)=$(e^{\theta}+e^{-\theta})/2$	(-∞,∞)	[1,∞)
Hyperbolic Tangent Function	tanh(θ)=$(e^{\theta}-e^{-\theta})/(e^{\theta}+e^{-\theta})$	(-∞,∞)	(-1,1)
Hyperbolic Cosecant Function	csch(θ)=$2/(e^{\theta}-e^{-\theta})$	(-∞,0)∪(0,∞)	(-∞,0)∪(0,∞)

Hyperbolic Functions Definitions (cont)

Hyperbolic Secant Function	sech(θ)=2/($e^{θ}$+$e^{-θ}$)	(-∞,∞)	(0,1]
Hyperbolic Cotangent Function	coth(θ)=($e^{θ}$+$e^{-θ}$)/($e^{θ}$-$e^{-θ}$)	(-∞,0)∪(0,∞)	(-∞,-1)∪(1,∞)
Inverse Hyperbolic Sine Function	arcsinh(x)=ln(x+√(x^2+1))	(-∞,∞)	(-∞,∞)
Inverse Hyperbolic Cosine Function	arccosh(x)=ln(x+√(x^2-1))	[1,∞)	[0,∞)
Inverse Hyperbolic Tangent Function	arctanh(x)=½ln((1+x)/(1-x))	(-1,1)	(-∞,∞)
Inverse Hyperbolic Cosecant Function	arccsch(x)=ln((1±√(1+x^2))/x)	(-∞,0)∪(0,∞)	(-∞,0)∪(0,∞)
Inverse Hyperbolic Secant Function	arcsech(x)=ln((1+√(1-x^2))/θ)	(0,1]	[0,∞)
Inverse Hyperbolic Cotangent Function	arccoth(x)=½ln((x+1)/(x-1))	(-∞,-1)∪(1,∞)	(-∞,0)∪(0,∞)
Hyperbolic Euler Relation	$e^{±θ}$=cosh(θ)±sinh(θ)		
De Moivre's Theorem (Hyperbolic)	$e^{nθ}$=(cosh(θ)+sinh(θ))^{n}=cosh(nθ)+sinh(nθ))		

n ∈ $\mathbb{N}_1$ = {1,2,3,4,5,...}

Complex Definitions

Name	Complex Relation	Circular-Hyperbolic Relation
Complex Sine	sin(z)=(e^{iz}-e^{-iz})/2i	sin(z)=-isinh(iz)
Complex Cosine	cos(z)=(e^{iz}+e^{-iz})/2	cos(z)=cosh(iz)
Complex Tangent	tan(z)=-i(e^{iz}-e^{-iz})/(e^{iz}+e^{-iz})	tan(z)=-itanh(iz)
Complex Cosecant	csc(z)=2i/(e^{iz}-e^{-iz})	csc(z)=icsch(iz)
Complex Secant	sec(z)=2/(e^{iz}+e^{-iz})	sec(z)=sech(iz)
Complex Cotangent	cot(z)=i(e^{iz}+e^{-iz})/(e^{iz}-e^{-iz})	cot(z)=icoth(iz)
Complex Inverse Sine	arcsin(z)=-iln(iz±√(1-z^2))	arcsin(z)=-iarcsinh(iz)
Complex Inverse Cosine	arccos(z)=-iln(z±i√(1-z^2))	arccos(z)=±iarccosh(z)

Complex Definitions (cont)

Complex Inverse Tangent	arctan(z)=(i/2)ln((i+z)/(i-z))	arctan(z)=-iarctanh(iz)
Complex Inverse Cosecant	arccsc(z)=-iln((i+√(z^2-1))/z)	arccsc(z)=iarccsch(iz)
Complex Inverse Secant	arcsec(z)=-iln((1+√(1-z^2))/z)	arcsec(z)=±iarcsech(z)
Complex Inverse Cotangent	arccot(z)=-(i/2)ln((z+i)/(z-i))	arccot(z)=±iarccoth(iz)
Complex Hyperbolic Sine	None	sinh(z)=-isin(iz)
Complex Hyperbolic Cosine	None	cosh(z)=cos(iz)
Complex Hyperbolic Tangent	None	tanh(z)=-itan(iz)
Complex Hyperbolic Cosecant	None	csch(z)=icsc(iz)
Complex Hyperbolic Secant	None	sech(z)=sec(iz)
Complex Hyperbolic Cotangent	None	coth(z)=icot(iz)
Complex Inverse Hyperbolic Sine	None	arcsinh(z)=-iarcsin(iz)
Complex Inverse Hyperbolic Cosine	None	arccosh(z)=±iarccos(z)
Complex Inverse Hyperbolic Tangent	None	arctanh(z)=-iarctan(iz)
Complex Inverse Hyperbolic Cosecant	None	arccsch(z)=-iarccsc(iz)
Complex Inverse Hyperbolic Secant	None	arcsech(z)=±iarcsec(z)
Complex Inverse Hyperbolic Cotangent	None	arccoth(z)=-iarccot(iz)

i=√(-1)

z is a complex variable of the form a+bi, where a and b are real numbers, and i is the imaginary number

Circular Functions Unit Circle Values

θ (Radians)	**θ (Degrees)**	***sin(θ)***	***cos(θ)***	***tan(θ)***	csc(θ)	sec(θ)	cot(θ)
0	**0°**	***0***	***1***	***0***	undefined	1	undefined
π/6	**30°**	***1/2***	***√3/2***	***√3/3***	2	2√3/3	√3
π/4	**45°**	***√2/2***	***√2/2***	***1***	√2	√2	1

Circular Functions Unit Circle Values (cont)

π/3	**60°**	***√3/2***	***1/2***	***√3***	2√3/3	2	√3/3
π/2	**90°**	***1***	***0***	***undefined***	1	undefined	0
2π/3	**120°**	√3/2	-1/2	-√3	2√3/3	-2	-√3/3
3π/4	**135°**	√2/2	-√2/2	-1	√2	-√2	-1
5π/6	**150°**	1/2	-√3/2	-√3/3	2	-2√3/3	-√3
π	**180°**	0	-1	0	undefined	-1	undefined
7π/6	**210°**	-1/2	-√3/2	√3/3	-2	-2√3/3	√3
5π/4	**225°**	-√2/2	-√2/2	1	-√2	-√2	1
4π/3	**240°**	-√3/2	-1/2	√3	-2√3/3	-2	√3/3
3π/2	**270°**	-1	0	undefined	-1	undefined	0
5π/3	**300°**	-√3/2	1/2	-√3	-2√3/3	2	-√3/3
7π/4	**315°**	-√2/2	√2/2	-1	-√2	√2	-1
11π/6	**330°**	-1/2	√3/2	-√3/3	-2	2√3/3	-√3
2π	**360°**	0	1	0	undefined	1	undefined

The coordinates (cos(θ),sin(θ)) represent x and y coordinates of θ on the unit circle $x^2+y^2=1$

Circular-Inverse Compositional Identities

Composition	**sin(x)**	**cos(x)**	**tan(x)**
arcsin(x)	x	$\sqrt{(1-x^2)}$	$x/\sqrt{(1-x^2)}$
arccos(x)	$\sqrt{(1-x^2)}$	x	$\sqrt{(1-x^2)}/x$
arctan(x)	$x/\sqrt{(1+x^2)}$	$1/\sqrt{(1+x^2)}$	x
arccsc(x)	1/x	$\sqrt{(x^2-1)}/\lvert x\rvert$	$\pm 1/\sqrt{(x^2-1)}$
arcsec(x)	$\sqrt{(x^2-1)}/\lvert x\rvert$	1/x	$\pm\sqrt{(x^2-1)}$
arccot(x)	$1/\sqrt{(1+x^2)}$	$x/\sqrt{(1+x^2)}$	1/x

Each composition is valid on different domains

Circular Quotient & Reciprocal Identities

Tangent Quotient	tan(θ)=sin(θ)/cos(θ)
Cotangent Quotient	cot(θ)=cos(θ)/sin(θ)
Sine Reciprocal	sin(θ)=1/csc(θ)
Cosine Reciprocal	cos(θ)=1/sec(θ)
Tangent Reciprocal	tan(θ)=1/cot(θ)
Cosecant Reciprocal	csc(θ)=1/sin(θ)
Secant Reciprocal	sec(θ)=1/cos(θ)
Cotangent Reciprocal	cot(θ)=1/tan(θ)

All the following identities are true for values that do not cause division by zero

Cofunctional Phase Shift Properties

Sine Complimentary	sin(θ)=cos(π/2-θ)
Sine Supplementary	sin(θ)=sin(π-θ)
Cosine Complimentary	cos(θ)=sin(π/2-θ)
Cosine Supplementary	cos(θ)=-cos(π-θ)

Cofunctional Phase Shift Properties (cont)

Tangent Complimentary	tan(θ)=cot(π/2-θ)
Tangent Supplementary	tan(θ)=-tan(πn-θ)
Cosecant Complimentary	csc(θ)=sec(π/2-θ)
Cosecant Supplementary	csc(θ)=csc(π-θ)
Secant Complimentary	sec(θ)=csc(π/2-θ)
Secant Supplementary	sec(θ)=-sec(π-θ)
Cotangent Complimentary	cot(θ)=tan(π/2-θ)
Cotangent Supplementary	cot(θ)=-cot(πn-θ)

$n \in \mathbb{N}_1 = \{1,2,3,4,5,...\}$

Periodicity Properties

Sine Periodicity	sin(θ)=sin(θ±2πn)
Cosine Periodicity	cos(θ)=cos(θ±2πn)
Tangent Periodicity	tan(θ)=tan(θ±πn)
Cosecant Periodicity	csc(θ)=csc(θ±2πn)
Secant Periodicity	sec(θ)=sec(θ±2πn)

Periodicity Properties (cont)

Cotangent Periodicity	cot(θ)=cot(θ±πn)

$n \in \mathbb{N}_1 = \{1,2,3,4,5,...\}$

Circular Parity Properties

Sine Odd	sin(-θ)=-sin(θ)
Cosine Even	cos(-θ)=cos(θ)
Tangent Odd	tan(-θ)=-tan(θ)
Cosecant Odd	csc(-θ)=-csc(θ)
Secant Even	sec(-θ)=sec(θ)
Cotangent Odd	cot(-θ)=-cot(θ)

Circular Pythagorean Identities

Sine Pythagorean	$\sin^2(\theta)=1-\cos^2(\theta)$
Cosine Pythagorean	$\cos^2(\theta)=1-\sin^2(\theta)$
Sine-Cosine Pythagorean	$\sin^2(\theta)+\cos^2(\theta)=1$
Secant Pythagorean	$\tan^2(\theta)+1=\sec^2(\theta)$
Tangent Pythagorean	$\tan^2(\theta)=\sec^2(\theta)-1$

Circular Pythagorean Identities (cont)

Secant-Tangent Pythagorean	$\sec^2(\theta)-\tan^2(\theta)=1$
Cosecant Pythagorean	$1+\cot^2(\theta)=\csc^2(\theta)$
Cotangent Pythagorean	$\cot^2(\theta)=\csc^2(\theta)-1$
Cosecant-Cotangent Pythagorean	$\csc^2(\theta)-\cot^2(\theta)=1$

The last two triplets of Pythagorean Identities are obtained by dividing all the terms of the original Sine-Cosine Identity by $\sin^2(\theta)$ or $\cos^2(\theta)$

(C) Half/Multiple-Angle Identities

Sine Half-Angle	sin(θ/2)=±√(½(1-cos(θ)))
Cosine Half-Angle	cos(θ/2)=±√(½(1+cos(θ)))
Tangent Half-Angle 1	tan(θ/2)=±√((1-cos(θ))/(1+cos(θ)))
Tangent Half-Angle 2	tan(θ/2)=(1-cos(θ))/sin(θ)
Tangent Half-Angle 3	tan(θ/2)=sin(θ)/(1+cos(θ))

(C) Half/Multiple-Angle Identities (cont)

Sine Double-Angle 1	sin(2θ)=2sin(θ)-cos(θ)
Sine Double-Angle 2	$\sin(2\theta)=2\tan(\theta)/(1+\tan^2(\theta))$
Cosine Double-Angle 1	$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$
Cosine Double-Angle 2	$\cos(2\theta)=2\cos^2(\theta)-1$
Cosine Double-Angle 3	$\cos(2\theta)=1-2\sin^2(\theta)$
Cosine Double-Angle 4	$\cos(2\theta)=(1-\tan^2(\theta))/(-1+\tan^2(\theta))$
Tangent Double-Angle 1	$\tan(2\theta)=2\tan(\theta)/(1-\tan^2(\theta))$
Tangent Double-Angle 2	tan(2θ)=2/(cot(-θ)-tan(θ))
Sine Triple-Angle	$\sin(3\theta)=3\sin(\theta)-4\sin^3(\theta)$
Cosine Triple-Angle	$\cos(3\theta)=4\cos^3(\theta)-3\cos(\theta)$

(C) Half/Multiple-Angle Identities (cont)

Tangent Triple-Angle	$(3\tan(\theta)-\tan^3(\theta))/(1-3\tan^2(\theta))$

Sine Multiple-Angle Formula: $\sin(n\theta)=\sum^n_{k=0} \binom{n}{k}\cos^k(\theta)\sin^{n-k}(\theta)\sin((\pi/2)(n-k))$
Cosine Multiple-Angle Formula: $\cos(n\theta)=\sum^n_{k=0} \binom{n}{k}\cos^k(\theta)\sin^{n-k}(\theta)\cos((\pi/2)(n-k))$

Circular Sum/Difference/Product Identities

Sine Sum/Difference	sin(θ±φ)	sin(θ)cos(φ)±cos(θ)sin(φ)
Sine Sum-Product	sin(θ)±sin(φ)	2sin((θ±φ)/2)cos((θ∓φ)/2)
Sine Product-Sum	sin(θ)sin(φ)	½(cos(θ-φ)-cos(θ+φ))
Cosine Sum/Difference	cos(θ±φ)	cos(θ)cos(φ)∓sin(θ)sin(φ)
Cosine Sum-Product	cos(θ)±cos(φ)	2cos((θ±φ)/2)cos((θ∓φ)/2)

Circular Sum/Difference/Product Identities (cont)

Cosine Product-Sum	cos(θ)cos(φ)	½(cos(θ-φ)+cos(θ+φ))
Sine-Cosine Product-Sum	sin(θ)cos(φ)	½(sin(θ-φ)+sin(θ+φ))
Tangent Sum/Difference	tan(θ±φ)	(tan(θ)±tan(φ))/(1∓tan(θ)tan(φ))
Tangent Sum	tan(θ)±tan(φ)	sin(θ±φ)/(cos(θ)cos(φ))
Tangent Product	tan(θ)tan(φ)	(tan(θ)+tan(φ))/(cot(θ)+cot(φ))
Tangent-Cotangent Product	tan(θ)cot(φ)	(tan(θ)+cot(φ))/(cot(θ)+tan(φ))

Law of Sines/Cosines/Tangents

Law of Sines 1	sin(α)/a=sin(β)/b=sin(γ)/c
Law of Sines 2	a/sin(α)=b/sin(β)=c/sin(γ)
Law of Cosines 1	$a^2=b^2+c^2-2bc\cos(\alpha)$

Law of Sines/Cosines/Tangents (cont)

Law of Cosines 2	$b^2=a^2+c^2-2ac\cos(\beta)$
Law of Cosines 3	$c^2=a^2+b^2-2ab\cos(\gamma)$
Law of Tangents 1	(a-b)/(a+b)=tan((α-β)/2)/tan((α+β)/2)
Law of Tangents 2	(b-c)/(b+c)=tan((β-γ)/2)/tan((β+γ)/2)
Law of Tangents 3	(c-a)/(c+a)=tan((γ-α)/2)/tan((γ+α)/2)

Side lengths a, b, and c are opposite of the angles α, β, and γ, respectively.

Measurements And Formulas

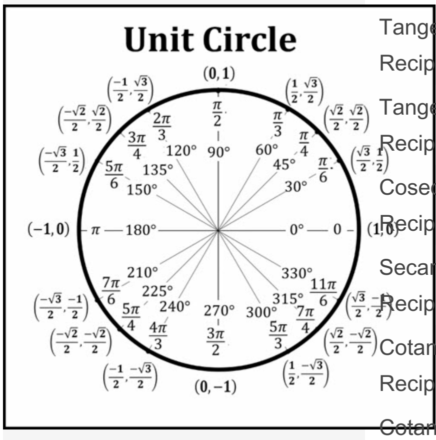
Radians-Degrees	1 radian=180/π degrees; 1=(180/π)°
Degrees-Radians	1 degree=π/180 radians; 1°=π/180 radians
Degrees, Minutes, and Seconds (DMS)	1 degree=60 minutes=3600 seconds;1°=60'=3600"

Measurements And Formulas (cont)

Arc Length/Angular Displacement	s=rθ units
Sector Area	½r^2θ $units^2$
Area of a Triangle	A_T=½bh $units^2$
Area of a Circle	A_C=πr^2 $units^2$
Pythagorean Theorem	$a^2+b^2=c^2$

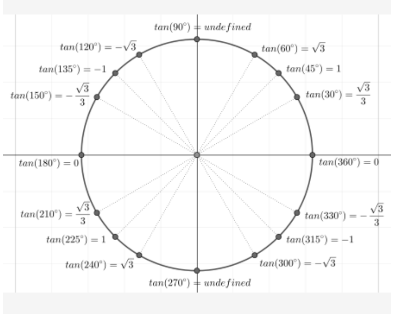
Radians are unitless
a, b, and c are side lengths of a right-triangle

Sine and Cosine Unit Circle

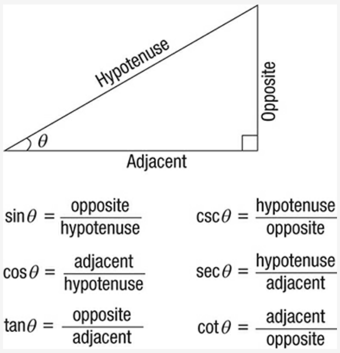


$x^2+y^2=1$

Tangent Unit Circle



Right-Triangle Relations



Circular-Inverse Reciprocal Identities

Sine Reciprocal	arcsin(1/x)=arccsc(x)
Cosine Reciprocal	arccos(1/x)=arcsec(x)
Tangent Reciprocal 1	arctan(1/x)=arccot(x), x>0
Tangent Reciprocal 2	arctan(1/x)=arccot(x)-π, x<0
Cosecant Reciprocal	arccsc(1/x)=arcsin(x)
Secant Reciprocal	arcsec(1/x)=arccos(x)
Cotangent Reciprocal 1	arccot(1/x)=arctan(x), x>0
Cotangent Reciprocal 2	arccot(1/x)=arctan(x)+π, x<0

Circular-Inverse Complimentary Identities

Sine Complimentary	arcsin(x)=π/2-arccos(x)
Cosine Complimentary	arccos(x)=π/2-arcsin(x)
Tangent Complimentary	arctan(x)=π/2-arccot(x)
Cosecant Complimentary	arccsc(x)=π/2-arcsec(x)
Secant Complimentary	arcsec(x)=π/2-arccsc(x)
Cotangent Complimentary	arccot(x)=π/2-arctan(x)

Circular-Inverse Negative Input Identities

Sine Odd	arcsin(-x)=-arcsin(x)
Cosine Translation	arccos(-x)=π-arccos(x)
Tangent Odd	arctan(-x)=-arctan(x)
Cosecant Odd	arccsc(-x)=-arccsc(x)
Secant Translation	arcsec(-x)=π-arcsec(x)
Cotangent Translation	arccot(-x)=π-arccot(x)

(CI) Half/Multiple Substitution Identities

Half Sine Substitution 1	½arcsin(x)=arcsin(√(1+x)/2))-π/4
Half Sine Substitution 2	½arcsin(x)=π/4-arcsin(√(1-x)/2))
Half Cosine Substitution 1	½arccos(x)=arccos(√((1+x)/2))
Half Cosine Substitution 2	½arccos(x)=π/2-arccos(√(1-x)/2))
Double Sine Substitution	2arcsin(x)=arcsin(2x√(1-x^2))
Double Cosine Substitution	2arccos(x)=arccos(2x^2-1)
Double Tangent Substitution 1	2arctan(x)=arcsin(2x/(1+x^2)), *|x|≤1*
Double Tangent Substitution 2	2arctan(x)=arccos((1-x^2)/(1+x^2)), *x≥0*
Double Tangent Substitution 3	2arctan(x)=arctan(2x/(1-x^2)), *|x|<1*

(CI) Half/Multiple Substitution Identities (cont)

Triple Sine Substitution	$3\arcsin(x)=\arcsin(3x-4x^3)$
Triple Cosine Substitution	$3\arccos(x)=\arccos(4x^3-3x)$
Triple Tangent Substitution	$3\arctan(x)=\arctan((3x-x^3)/(1-3x^2))$

Each identity is valid for the proper domains of the functions

Circular-Inverse Sum/Difference Identities

Sine Sum/Difference	$\arcsin(x)\pm\arcsin(y)=\arcsin(x\sqrt{(1-y^2)}\pm y\sqrt{(1-x^2)})$
Cosine Sum/Difference	$\arccos(x)\pm\arccos(y)=\arccos(xy\mp\sqrt{(1-x^2)}\sqrt{(1-y^2)})$
Cosine-Sine Sum/Difference	$\arccos(x)\pm\arcsin(y)=\arccos(x\sqrt{(1-y^2)}\mp y\sqrt{(1-x^2)})$
Tangent Sum/Difference	$\arctan(x)\pm\arctan(y)=\arctan((x\pm y)/(1\mp xy))$, $1\mp xy\neq0$

(H) Quotient & Reciprocal Identities

Tangent Quotient	$\tanh(\theta)=\sinh(\theta)/\cosh(\theta)$
Cotangent Quotient	$\coth(\theta)=\cosh(\theta)/\sinh(\theta)$
Sine Reciprocal	$\sinh(\theta)=1/\text{csch}(\theta)$
Cosine Reciprocal	$\cosh(\theta)=1/\text{sech}(\theta)$
Tangent Reciprocal	$\tanh(\theta)=1/\coth(\theta)$
Cosecant Reciprocal	$\text{csch}(\theta)=1/\sinh(\theta)$
Secant Reciprocal	$\text{sech}(\theta)=1/\cosh(\theta)$
Cotangent Reciprocal	$\coth(\theta)=1/\tanh(\theta)$

All the following identities are true for values that do not cause division by zero

Hyperbolic Parity Properties

Sine Odd	$\sinh(-\theta)=-\sinh(\theta)$
Cosine Even	$\cosh(-\theta)=-\cosh(\theta)$
Tangent Odd	$\tanh(-\theta)=-\tanh(\theta)$

Hyperbolic Parity Properties (cont)

Cosecant Odd	$\text{csch}(-\theta)=-\text{csch}(\theta)$
Secant Even	$\text{sech}(-\theta)=-\text{sech}(\theta)$
Cotangent Odd	$\coth(-\theta)=-\coth(\theta)$

(HI) Negative Input Identities

Inverse Sine Odd	$\text{arcsinh}(-x)=-\text{arcsinh}(x)$
Inverse Tangent Odd	$\text{arctanh}(-x)=-\text{arctanh}(x)$
Inverse Cosecant Odd	$\text{arccsch}(-x)=-\text{arccsch}(x)$
Inverse Cotangent Odd	$\text{arccoth}(-x)=-\text{arccoth}(x)$

Hyperbolic Pythagorean Identities

Sine Pythagorean	$\sinh^2(\theta)=\cosh^2(\theta)-1$
Cosine Pythagorean	$\cosh^2(\theta)=\sinh^2(\theta)+1$
Sine-Cosine Pythagorean	$\cosh^2(\theta)-\sinh^2(\theta)=1$
Secant Pythagorean	$1-\tanh^2(\theta)=\text{sech}^2(\theta)$
Tangent Pythagorean	$\tanh^2(\theta)=1-\text{sech}^2(\theta)$
Secant-Tangent Pythagorean	$\text{sech}^2(\theta)+\tanh^2(\theta)=1$

Hyperbolic Pythagorean Identities (cont)

Cosecant Pythagorean	$\coth^2(\theta)-1=\text{csch}^2(\theta)$
Cotangent Pythagorean	$\coth^2(\theta)=\text{csch}^2(\theta)+1$
Cosecant-Cotangent Pythagorean	$\coth^2(\theta)-\text{csch}^2(\theta)=1$

The last two triplets of Hyperbolic Pythagorean Identities are obtained by dividing all the terms of the original Sine-Cosine Identity by $\sinh^2(\theta)$ or $\cosh^2(\theta)$

(H) Half-Angle & Multiple-Angle Identities

Sine Half-Angle	$\sinh(\theta/2)=\sqrt{(½(\cosh(\theta)-1))}$
Cosine Half-Angle	$\cosh(\theta/2)=\sqrt{(½(\cosh(\theta)+1))}$
Tangent Half-Angle 1	$\tanh(\theta/2)=\sqrt{((\cosh(\theta)-1)/(\cosh(\theta)+1))}$
Tangent Half-Angle 2	$\tanh(\theta/2)=(\cosh(\theta)-1)/\sinh(\theta)$
Tangent Half-Angle 3	$\tanh(\theta/2)=\sinh(\theta)/(\cosh(\theta)+1)$

(H) Half-Angle & Multiple-Angle Identities (cont)

Sine Double-Angle 1	$\sinh(2\theta)=2\sinh(\theta)\cosh(\theta)$
Sine Double-Angle 2	$\sinh(2\theta)=2\tanh(\theta)/(1-\tanh^2(\theta))$
Cosine Double-Angle 1	$\cosh(2\theta)=\cosh^2(\theta)+\sinh^2(\theta)$
Cosine Double-Angle 2	$\cosh(2\theta)=2\cosh^2(\theta)-1$
Cosine Double-Angle 3	$\cosh(2\theta)=1+2\sinh^2(\theta)$
Cosine Double-Angle 4	$\cosh(2\theta)=(1+\tanh^2(\theta))/(1-\tanh^2(\theta))$
Tangent Double Angle 1	$\tanh(2\theta)=2\tanh(\theta)/(1+\tanh^2(\theta))$
Tangent Double Angle 2	$\tanh(2\theta)=2/(\coth(\theta)+\tanh(\theta))$
Sine Triple-Angle	$\sinh(3\theta)=3\sinh(\theta)+4\sinh^3(\theta)$
Cosine Triple-Angle	$\cosh(3\theta)=4\cosh^3(\theta)-3\cosh(\theta)$

(H) Half-Angle & Multiple-Angle Identities (cont)

Tangent Triple-Angle	$\tanh(3\theta)=(3\tan(\theta)+\tan^3(\theta))/(1+3\tan^2(\theta))$

(H) Sum/Difference/Product Identities

Sine Sum/Difference	$\sinh(\theta\pm\varphi)$	$\sinh(\theta)\cosh(\varphi)\pm\cosh(\theta)\sinh(\varphi)$
Sine Sum-Product	$\sinh(\theta)\pm\sinh(\varphi)$	$2\sinh((\theta\pm\varphi)/2)\cosh((\theta\mp\varphi)/2)$
Sine Product-Sum	$\sinh(\theta)\sinh(\varphi)$	$½(\cosh(\theta+\varphi)-\cosh(\theta-\varphi))$
Cosine Sum/Difference	$\cosh(\theta\pm\varphi)$	$\cosh(\theta)\cosh(\varphi)\mp\sinh(\theta)\sinh(\varphi)$
Cosine Sum-Product 1	$\cosh(\theta)+\cosh(\varphi)$	$2\cosh((\theta+\varphi)/2)\cosh((\theta-\varphi)/2)$

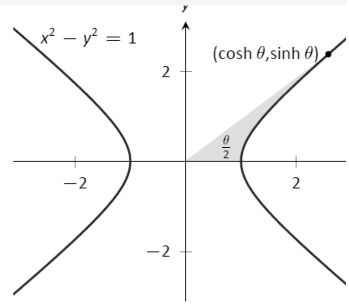
(H) Sum/Difference/Product Identities (cont)

Cosine Sum-Product 2	$\cosh(\theta)-\cosh(\varphi)$	$2\sinh((\theta+\varphi)/2)\sinh((\theta-\varphi)/2)$
Cosine Product-Sum	$\cosh(\theta)\cosh(\varphi)$	$½(\cosh(\theta+\varphi)+\cosh(\theta-\varphi))$
Sine-Cosine Product-Sum	$\sinh(\theta)\cosh(\varphi)$	$½(\sinh(\theta+\varphi)+\sinh(\theta-\varphi))$
Tangent Sum/Difference	$\tanh(\theta\pm\varphi)$	$(\tanh(\theta)\pm\tanh(\varphi))/(1\pm\tanh(\theta)\tanh(\varphi))$
Tangent Sum	$\tanh(\theta)\pm\tanh(\varphi)$	$\sinh(\theta\pm\varphi)/(\cosh(\theta)\cosh(\varphi))$
Tangent Product	$\tanh(\theta)\tanh(\varphi)$	$(\tanh(\theta)+\tanh(\varphi))/(\coth(\theta)+\coth(\varphi))$

(H) Sum/Difference/Product Identities (cont)

Tangent-Cotangent Product	$\tanh(\theta)\coth(\varphi)$	$(\tanh(\theta)+\coth(\varphi))/(\coth(\theta)+\tanh(\varphi))$

Unit Hyperbola



$x^2-y^2=1$