Circular Functions Definitions

Name	Right-Triangle Definition	Domain	Range
Sine Function	sin(θ)=o/h	(-∞,∞)	[-1,1]
Cosine Function	cos(θ)=a/h	(-∞,∞)	[-1,1]
Tangent Function	tan(θ)=o/a	{θ|θ≠π/2±πn}	(-∞,∞)
Cosecant Function	csc(θ)=h/o	{θ|θ≠±πn}	(-∞,-1]∪[1,∞)
Secant Function	sec(θ)=h/a	{θ|θ≠π/2±πn}	(-∞,-1]∪[1,∞)
Cotangent Function	cot(θ)=a/o	{θ|θ≠±πn}	(-∞,∞)
Inverse Sine Function	arcsin(o/h)=θ	[-1,1]	[-π/2,π/2]
Inverse Cosine Function	arccos(a/h)=θ	[-1,1]	[0,π]
Inverse Tangent Function	arctan(o/a)=θ	(-∞,∞)	(-1,1)
Inverse Cosecant Function	arccsc(h/o)=θ	(-∞,-1)∪(1,∞)	[-π/2,0)∪(0,π/2]
Inverse Secant Function	arcsec(h/a)=θ	(-∞,-1)∪(1,∞)	[0,π/2)∪(π/2,π]
Inverse Cotangent Function	arccot(a/o)=θ	(-∞,∞)	(0,1)
Circular Euler Relation	$e^{\pm i\theta}$=cos(θ)±isin(θ)		
De Moivre's Theorem	$e^{in\theta}$=$(\cos(\theta)+i\sin(\theta))^n$=cos(nθ)+isin(nθ)		

$n \in \mathbb{N}_1$ = {1,2,3,4,5,...}

"h" is the "hypotenuse" leg of a right triangle. It is directly across the right (90°) angle, and it has the longest length of the three sides.

"o" is the "opposite" leg of a right triangle. It is directly across the chosen angle θ.

"a" is the "adjacent" leg of a right triangle. It is the leg that is neither the hypotenuse leg, nor the opposite leg.

By the Pythagorean theorem, $o^2+a^2=h^2$

Hyperbolic Functions Definitions

Name	Exponential Definition	Domain	Range
Hyperbolic Sine Function	$\sinh(\theta)=(e^{\theta}-e^{-\theta})/2$	(-∞,∞)	(-∞,∞)
Hyperbolic Cosine Function	$\cosh(\theta)=(e^{\theta}+e^{-\theta})/2$	(-∞,∞)	[1,∞)
Hyperbolic Tangent Function	$\tanh(\theta)=(e^{\theta}-e^{-\theta})/(e^{\theta}+e^{-\theta})$	(-∞,∞)	(-1,1)
Hyperbolic Cosecant Function	$\text{csch}(\theta)=2/(e^{\theta}-e^{-\theta})$	(-∞,0)∪(0,∞)	(-∞,0)∪(0,∞)
Hyperbolic Secant Function	$\text{sech}(\theta)=2/(e^{\theta}+e^{-\theta})$	(-∞,∞)	(0,1]

Hyperbolic Functions Definitions (cont)

Hyperbolic Cotangent Function	coth(θ)=($e^{θ}$+$e^{-θ}$)/($e^{θ}$-$e^{-θ}$)	(-∞,0)∪(0,∞)	(-∞,-1)∪(1,∞)
Inverse Hyperbolic Sine Function	arcsinh(x)=ln(x+√(x^2+1))	(-∞,∞)	(-∞,∞)
Inverse Hyperbolic Cosine Function	arccosh(x)=ln(x+√(x^2-1))	[1,∞)	[0,∞)
Inverse Hyperbolic Tangent Function	arctanh(x)=½ln((1+x)/(1-x))	(-1,1)	(-∞,∞)
Inverse Hyperbolic Cosecant Function	arccsch(x)=ln((1±√(1+x^2))/x)	(-∞,0)∪(0,∞)	(-∞,0)∪(0,∞)
Inverse Hyperbolic Secant Function	arcsech(x)=ln((1+√(1-x^2))/x)	(0,1]	[0,∞)
Inverse Hyperbolic Cotangent Function	arccoth(x)=½ln((x+1)/(x-1))	(-∞,-1)∪(1,∞)	(-∞,0)∪(0,∞)
Hyperbolic Euler Relation	$e^{±θ}$=cosh(θ)±sinh(θ)		
De Moivre's Theorem (Hyperbolic)	$e^{nθ}$=(cosh(θ)+sinh(θ)$)^n$=cosh(nθ)+sinh(nθ)		

n ∈ $\mathbb{N}_1$ = {1,2,3,4,5,...}

Complex Definitions

Name	Complex Relation	Circular-Hyperbolic Relation
Complex Sine	sin(z)=(e^{iz}-e^{-iz})/2i	sin(z)=-isinh(iz)
Complex Cosine	cos(z)=(e^{iz}+e^{-iz})/2	cos(z)=cosh(iz)
Complex Tangent	tan(z)=-i(e^{iz}-e^{-iz})/(e^{iz}+e^{-iz})	tan(z)=-itanh(iz)
Complex Cosecant	csc(z)=2i/(e^{iz}-e^{-iz})	csc(z)=icsch(iz)
Complex Secant	sec(z)=2/(e^{iz}+e^{-iz})	sec(z)=sech(iz)
Complex Cotangent	cot(z)=i(e^{iz}+e^{-iz})/(e^{iz}-e^{-iz})	cot(z)=icoth(iz)
Complex Inverse Sine	arcsin(z)=-iln(iz±√(1-z^2))	arcsin(z)=-iarcsinh(iz)
Complex Inverse Cosine	arccos(z)=-iln(z±i√(1-z^2))	arccos(z)=±iarccosh(z)
Complex Inverse Tangent	arctan(z)=(i/2)ln((i+z)/(i-z))	arctan(z)=-iarctanh(iz)
Complex Inverse Cosecant	arccsc(z)=-iln((i+√(z^2-1))/z)	arccsc(z)=iarccsch(iz)
Complex Inverse Secant	arcsec(z)=-iln((1+√(1-z^2))/z)	arcsec(z)=±iarcsech(z)
Complex Inverse Cotangent	arccot(z)=-(i/2)ln((z+i)/(z-i))	arccot(z)=±iarccoth(iz)
Complex Hyperbolic Sine	None	sinh(z)=-isin(iz)

Complex Definitions (cont)

Complex Hyperbolic Cosine	None	cosh(z)=cos(iz)
Complex Hyperbolic Tangent	None	tanh(z)=-itan(iz)
Complex Hyperbolic Cosecant	None	csch(z)=icsc(iz)
Complex Hyperbolic Secant	None	sech(z)=sec(iz)
Complex Hyperbolic Cotangent	None	coth(z)=icot(iz)
Complex Inverse Hyperbolic Sine	None	arcsinh(z)=-iarcsin(iz)
Complex Inverse Hyperbolic Cosine	None	arccosh(z)=±iarccos(z)
Complex Inverse Hyperbolic Tangent	None	arctanh(z)=-iarctan(iz)
Complex Inverse Hyperbolic Cosecant	None	arccsch(z)=-iarccsc(iz)
Complex Inverse Hyperbolic Secant	None	arcsech(z)=±iarcsec(z)
Complex Inverse Hyperbolic Cotangent	None	arccoth(z)=-iarccot(iz)

i=√(-1)

z is a complex variable of the form a+bi, where a and b are real numbers, and i is the imaginary unit

Circular Functions Unit Circle Values

θ (Radians)	**θ (Degrees)**	***sin(θ)***	***cos(θ)***	***tan(θ)***	csc(θ)	sec(θ)	cot(θ)
0	**0°**	***0***	***1***	***0***	undefined	1	undefined
π/6	**30°**	***1/2***	***√3/2***	***√3/3***	2	2√3/3	√3
π/4	**45°**	***√2/2***	***√2/2***	***1***	√2	√2	1
π/3	**60°**	***√3/2***	***1/2***	***√3***	2√3/3	2	√3/3
π/2	**90°**	***1***	***0***	***undefined***	1	undefined	0
2π/3	**120°**	√3/2	-1/2	-√3	2√3/3	-2	-√3/3
3π/4	**135°**	√2/2	-√2/2	-1	√2	-√2	-1
5π/6	**150°**	1/2	-√3/2	-√3/3	2	-2√3/3	-√3
π	**180°**	0	-1	0	undefined	-1	undefined
7π/6	**210°**	-1/2	-√3/2	√3/3	-2	-2√3/3	√3
5π/4	**225°**	-√2/2	-√2/2	1	-√2	-√2	1
4π/3	**240°**	-√3/2	-1/2	√3	-2√3/3	-2	√3/3

Circular Functions Unit Circle Values (cont)

3π/2	**270°**	-1	0	undefined	-1	undefined	0
5π/3	**300°**	-√3/2	1/2	-√3	-2√3/3	2	-√3/3
7π/4	**315°**	-√2/2	√2/2	-1	-√2	√2	-1
11π/6	**330°**	-1/2	√3/2	-√3/3	-2	2√3/3	-√3
2π	**360°**	0	1	0	undefined	1	undefined

The coordinates (cos(θ), sin(θ)) represent x and y coordinates of θ on the unit circle $x^2+y^2=1$

Circular Compositional Identities

Composition	sin(x)	cos(x)	tan(x)
arcsin(x)	x	$\sqrt{(1-x^2)}$	$x/\sqrt{(1-x^2)}$
arccos(x)	$\sqrt{(1-x^2)}$	x	$\sqrt{(1-x^2)}/x$
arctan(x)	$x/\sqrt{(1+x^2)}$	$1/\sqrt{(1+x^2)}$	x
arccsc(x)	1/x	$\sqrt{(x^2-1)}/\lvert x\rvert$	$\pm1/\sqrt{(x^2-1)}$
arcsec(x)	$\sqrt{(x^2-1)}/\lvert x\rvert$	1/x	$\pm\sqrt{(x^2-1)}$
arccot(x)	$1/\sqrt{(1+x^2)}$	$x/\sqrt{(1+x^2)}$	1/x

Each composition is valid on different domains

Hyperbolic Compositional Identities

Composition	sinh(x)	cosh(x)	tanh(x)
arcsinh(x)	x	$\sqrt{(1+x^2)}$	$x/\sqrt{(1-x^2)}$
arccosh(x)	$\sqrt{(x^2-1)}$	x	$\sqrt{(x^2-1)}/x$
arctanh(x)	$x/\sqrt{(1-x^2)}$	$1/\sqrt{(1-x^2)}$	x
arccsch(x)	1/x	$\sqrt{(x^2+1)}/\lvert x\rvert$	$1/\sqrt{(x^2+1)}$
arcsech(x)	$\sqrt{(1-x^2)}/x$	1/x	$\sqrt{(1-x^2)}$
arccoth(x)	$x/\sqrt{(1-x^2)}$	$\lvert x\rvert/\sqrt{(x^2-1)}$	1/x

Each composition is valid on different domains

Circular Quotient & Reciprocal Identities

Cofunctional Phase Shift Properties (cont)

Cosecant Complimentary	csc(θ)=sec(π/2-θ)
Cosecant Supplementary	csc(θ)=csc(π-θ)
Secant Complimentary	sec(θ)=csc(π/2-θ)
Secant Supplementary	sec(θ)=-sec(π-θ)
Cotangent Complimentary	cot(θ)=tan(π/2-θ)
Cotangent Supplementary	cot(θ)=-cot(πn-θ)

$n \in \mathbb{N}_1 = \{1,2,3,4,5,...\}$

Periodicity Properties

Circular Parity Properties

Sine Odd	sin(-θ)=-sin(θ)
Cosine Even	cos(-θ)=cos(θ)
Tangent Odd	tan(-θ)=-tan(θ)
Cosecant Odd	csc(-θ)=-csc(θ)
Secant Even	sec(-θ)=sec(θ)
Cotangent Odd	cot(-θ)=-cot(θ)

Circular Pythagorean Identities

(C) Half/Multiple-Angle Identities (cont)

Tangent Quotient	$\tan(\theta)=\sin(\theta)/\cos(\theta)$
Cotangent Quotient	$\cot(\theta)=\cos(\theta)/\sin(\theta)$
Sine Reciprocal	$\sin(\theta)=1/\csc(\theta)$
Cosine Reciprocal	$\cos(\theta)=1/\sec(\theta)$
Tangent Reciprocal	$\tan(\theta)=1/\cot(\theta)$
Cosecant Reciprocal	$\csc(\theta)=1/\sin(\theta)$
Secant Reciprocal	$\sec(\theta)=1/\cos(\theta)$
Cotangent Reciprocal	$\cot(\theta)=1/\tan(\theta)$

All the following identities are true for values that do not cause division by zero

Cofunctional Phase Shift Properties

Sine Complimentary	$\sin(\theta)=\cos(\pi/2-\theta)$
Sine Supplementary	$\sin(\theta)=\sin(\pi-\theta)$
Cosine Complimentary	$\cos(\theta)=\sin(\pi/2-\theta)$
Cosine Supplementary	$\cos(\theta)=-\cos(\pi-\theta)$
Tangent Complimentary	$\tan(\theta)=\cot(\pi/2-\theta)$
Tangent Supplementary	$\tan(\theta)=-\tan(\pi n-\theta)$

Sine Periodicity	$\sin(\theta)=\sin(\theta\pm2\pi n)$
Cosine Periodicity	$\cos(\theta)=\cos(\theta\pm2\pi n)$
Tangent Periodicity	$\tan(\theta)=\tan(\theta\pm\pi n)$
Cosecant Periodicity	$\csc(\theta)=\csc(\theta\pm2\pi n)$
Secant Periodicity	$\sec(\theta)=\sec(\theta\pm2\pi n)$
Cotangent Periodicity	$\cot(\theta)=\cot(\theta\pm\pi n)$

$n \in \mathbb{N}_1 = \{1,2,3,4,5,...\}$

Sine-Cosine Pythagorean	$\sin^2(\theta)+\cos^2(\theta)=1$
Secant-Tangent Pythagorean	$\tan^2(\theta)+1=\sec^2(\theta)$
Cosecant-Cotangent Pythagorean	$1+\cot^2(\theta)=\csc^2(\theta)$

The last two Pythagorean Identities are obtained by dividing all the terms of the original Sine-Cosine Identity by $\cos^2(\theta)$ and $\sin^2(\theta)$, respectively

(C) Half/Multiple-Angle Identities

Sine Half-Angle	$\sin(\theta/2)=\pm\sqrt{(½(1-\cos(\theta)))}$
Cosine Half-Angle	$\cos(\theta/2)=\pm\sqrt{(½(1+\cos(\theta)))}$
Tangent Half-Angle 1	$\tan(\theta/2)=\pm\sqrt{((1-\cos(\theta))/(1+\cos(\theta)))}$
Tangent Half-Angle 2	$\tan(\theta/2)=(1-\cos(\theta))/\sin(\theta)$
Tangent Half-Angle 3	$\tan(\theta/2)=\sin(\theta)/(1+\cos(\theta))$
Sine Double-Angle 1	$\sin(2\theta)=2\sin(\theta)\cos(\theta)$
Sine Double-Angle 2	$\sin(2\theta)=2\tan(\theta)/(1+\tan^2(\theta))$
Cosine Double-Angle 1	$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$
Cosine Double-Angle 2	$\cos(2\theta)=2\cos^2(\theta)-1$
Cosine Double-Angle 3	$\cos(2\theta)=1-2\sin^2(\theta)$
Cosine Double-Angle 4	$\cos(2\theta)=(1-\tan^2(\theta))/(-1+\tan^2(\theta))$
Tangent Double-Angle 1	$\tan(2\theta)=2\tan(\theta)/(1-\tan^2(\theta))$
Tangent Double-Angle 2	$\tan(2\theta)=2/(\cot(\theta)-\tan(\theta))$
Sine Triple-Angle	$\sin(3\theta)=3\sin(\theta)-4\sin^3(\theta)$

(C) Half/Multiple-Angle Identities (cont)

Cosine Triple-Angle	$\cos(3\theta)=4\cos^3(\theta)-3\cos(\theta)$
Tangent Triple-Angle	$\tan(3\theta)=(3\tan(\theta)-\tan^3(\theta))/(1-3\tan^2(\theta))$

Sine Multiple-Angle Formula: $\sin(n\theta)=\sum^n_{k=0} (^n{}_k)\cos^k(\theta)\sin^{n-k}(\theta)\sin((\pi/2)(n-k))$

Cosine Multiple-Angle Formula: $\cos(n\theta)=\sum^n_{k=0} (^n{}_k)\cos^k(\theta)\sin^{n-k}(\theta)\cos((\pi/2)(n-k))$

All the following identities are true for values that do not cause division by zero

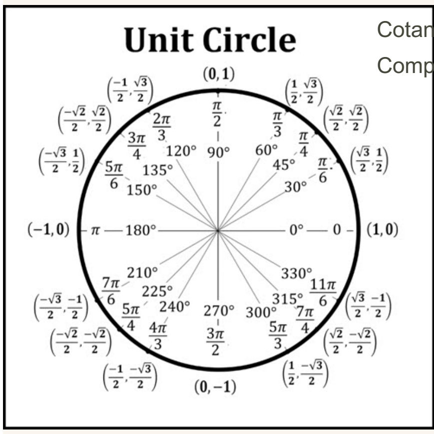
Circular Sum/Difference/Product Identities

Sine Sum/Difference	$\sin(\theta\pm\varphi)$	$\sin(\theta)\cos(\varphi)\pm\cos(\theta)\sin(\varphi)$
Sine Sum-Product	$\sin(\theta)\pm\sin(\varphi)$	$2\sin((\theta\pm\varphi)/2)\cos((\theta\mp\varphi)/2)$
Sine Product-Sum	$\sin(\theta)\sin(\varphi)$	$\frac{1}{2}(\cos(\theta-\varphi)-\cos(\theta+\varphi))$
Cosine Sum/Difference	$\cos(\theta\pm\varphi)$	$\cos(\theta)\cos(\varphi)\mp\sin(\theta)\sin(\varphi)$
Cosine Sum-Product	$\cos(\theta)\pm\cos(\varphi)$	$2\cos((\theta\pm\varphi)/2)\cos((\theta\mp\varphi)/2)$

Circular Sum/Difference/Product Identities (cont)

Cosine Product-Sum	$\cos(\theta)\cos(\varphi)$	$\frac{1}{2}(\cos(\theta-\varphi)+\cos(\theta+\varphi))$
Sine-Cosine Product-Sum	$\sin(\theta)\cos(\varphi)$	$\frac{1}{2}(\sin(\theta-\varphi)+\sin(\theta+\varphi))$
Tangent Sum/Difference	$\tan(\theta\pm\varphi)$	$(\tan(\theta)\pm\tan(\varphi))/(1\mp\tan(\theta)\tan(\varphi))$
Tangent Sum	$\tan(\theta)\pm\tan(\varphi)$	$\sin(\theta\pm\varphi)/(\cos(\theta)\cos(\varphi))$
Tangent Product	$\tan(\theta)\tan(\varphi)$	$(\tan(\theta)+\tan(\varphi))/(\cot(\theta)+\cot(\varphi))$
Tangent-Cotangent Product	$\tan(\theta)\cot(\varphi)$	$(\tan(\theta)+\cot(\varphi))/(\cot(\theta)+\tan(\varphi))$

Sine and Cosine Unit Circle



$x^2+y^2=1$

Circular-Inverse Reciprocal Identities

Sine Reciprocal	$\arcsin(1/x)=\text{arccsc}(x)$
Cosine Reciprocal	$\arccos(1/x)=\text{arcsec}(x)$
Tangent Reciprocal 1	$\arctan(1/x)=\text{arccot}(x)$, *$x>0$*
Tangent Reciprocal 2	$\arctan(1/x)=\text{arccot}(x)-\pi$, *$x<0$*
Cosecant Reciprocal	$\text{arccsc}(1/x)=\arcsin(x)$
Secant Reciprocal	$\text{arcsec}(1/x)=\arccos(x)$
Cotangent Reciprocal 1	$\text{arccot}(1/x)=\arctan(x)$, *$x>0$*
Cotangent Reciprocal 2	$\text{arccot}(1/x)=\arctan(x)+\pi$, *$x<0$*

Circular-Inverse Complimentary Identities

Sine Complimentary	$\arcsin(x)=\pi/2-\arccos(x)$
Cosine Complimentary	$\arccos(x)=\pi/2-\arcsin(x)$
Tangent Complimentary	$\arctan(x)=\pi/2-\text{arccot}(x)$
Cosecant Complimentary	$\text{arccsc}(x)=\pi/2-\text{arcsec}(x)$
Secant Complimentary	$\text{arcsec}(x)=\pi/2-\text{arccsc}(x)$
Cotangent Complimentary	$\text{arccot}(x)=\pi/2-\arctan(x)$

Circular-Inverse Negative Input Identities

Sine Odd	$\arcsin(-x)=-\arcsin(x)$
Cosine Translation	$\arccos(-x)=\pi-\arccos(x)$
Tangent Odd	$\arctan(-x)=-\arctan(x)$
Cosecant Odd	$\text{arccsc}(-x)=-\text{arccsc}(x)$
Secant Translation	$\text{arcsec}(-x)=\pi-\text{arcsec}(x)$
Cotangent Translation	$\text{arccot}(-x)=\pi-\text{arccot}(x)$

(CI) Half/Multiple Substitution Identities

Half Sine Substitution 1	$\frac{1}{2}\arcsin(x)=\arcsin(\sqrt{}(1+x)/2))-\pi/4$
Half Sine Substitution 2	$\frac{1}{2}\arcsin(x)=\pi/4-\arcsin(\sqrt{}(1-x)/2))$
Half Cosine Substitution 1	$\frac{1}{2}\arccos(x)=\arccos(\sqrt{}((1+x)/2))$
Half Cosine Substitution 2	$\frac{1}{2}\arccos(x)=\pi/2-\arccos(\sqrt{}(1-x)/2))$
Double Sine Substitution	$2\arcsin(x)=\arcsin(2x\sqrt{}(1-x^2))$, *$\lvert x\rvert\le\pi/2$*
Double Cosine Substitution 1	$2\arccos(x)=\arccos(2x^2-1)$, *$x\ge0$*

(CI) Half/Multiple Substitution Identities (cont)

Double Cosine Substitution 2	2arccos(x)=2π-arccos($2x^2$-1), *x≤0*
Double Tangent Substitution 1	2arctan(x)=arcsin(2x/(1+x^2)), *|x|≤1*
Double Tangent Substitution 2	2arctan(x)=±arccos((1-x^2)/(1+x^2))
Double Tangent Substitution 3	2arctan(x)=arctan(2x/(1-x^2)), *|x|<1*
Triple Sine Substitution 1	3arcsin(x)=arcsin(3x-4x^3), *|x|≤½*
Triple Sine Substitution 2	3arcsin(x)=arcsin(4x^3-3x)±π, *|x|≥½*
Triple Cosine Substitution 1	3arccos(x)=arccos(3x-4x^3), *|x|≤½*
Triple Cosine Substitution 2	3arccos(x)=arccos(4x^3-3x)+π±π, *|x|≥½*
Triple Tangent Substitution 1	3arctan(x)=arctan((3x-x^3)/(1-3x^2)), *|x|≤√3/3*
Triple Tangent Substitution 2	3arctan(x)=arctan((3x-x^3)/(1-3x^2))±π, *|x|≥√3/3*

Circular-Inverse Sum/Difference Identities

Sine Sum/Difference	arcsin(x)±arcsin(y)=arcsin(x√(1-y^2)±y√(1-x^2))
Cosine Sum/Difference	arccos(x)±arccos(y)=arccos(xy∓√(1-x^2)√(1-y^2))
Cosine-Sine Sum/Difference	arccos(x)±arcsin(y)=arccos(x√(1-y^2)∓y√(1-x^2))
Tangent Sum/Difference	arctan(x)±arctan(y)=arctan((x±y)/(1∓xy)), *1∓xy≠0*

Law of Sines/Cosines/Tangents

Law of Sines 1	sin(α)/a=sin(β)/b=sin(γ)/c
Law of Sines 2	a/sin(α)=b/sin(β)=c/sin(γ)
Law of Cosines 1	a^2=b^2+c^2-2bccos(α)
Law of Cosines 2	b^2=a^2+c^2-2accos(β)
Law of Cosines 3	c^2=a^2+b^2-2abcos(γ)
Law of Tangents 1	(a-b)/(a+b)=tan((α-β)/2)/tan((α+β)/2)

Law of Sines/Cosines/Tangents (cont)

Law of Tangents 2	(b-c)/(b+c)=tan((β-γ)/2)/tan((β+γ)/2)
Law of Tangents 3	(c-a)/(c+a)=tan((γ-α)/2)/tan((γ+α)/2)

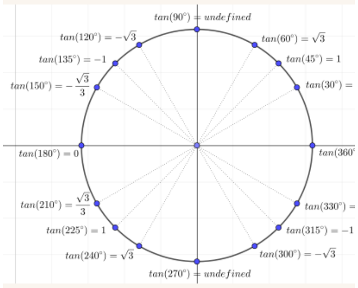
Side lengths a, b, and c are opposite of the angles α, β, and γ, respectively.

Measurements And Formulas

Radians-Degrees	1 radian=180/π degrees; 1=(180/π)°
Degrees-Radians	1 degree=π/180 radians; 1°=π/180 radians
Degrees, Minutes, and Seconds (DMS)	1 degree=60 minutes=3600 seconds;1°=60'=3600"
Arc Length/Angular Displacement	s=rθ units
Sector Area	½r^2θ units^2
Area of a Triangle	A_T=½bh units^2
Area of a Circle	A_C=πr^2 units^2
Pythagorean Theorem	a^2+b^2=c^2

Radians are unitless
a, b, and c are side lengths of a right-triangle

Tangent Unit Circle



(H) Quotient & Reciprocal Identities

Tangent Quotient	tanh(θ)=sinh(θ)/cosh(θ)
Cotangent Quotient	coth(θ)=cosh(θ)/sinh(θ)
Sine Reciprocal	sinh(θ)=1/csch(θ)
Cosine Reciprocal	cosh(θ)=1/sech(θ)
Tangent Reciprocal	tanh(θ)=1/coth(θ)
Cosecant Reciprocal	csch(θ)=1/sinh(θ)
Secant Reciprocal	sech(θ)=1/cosh(θ)
Cotangent Reciprocal	coth(θ)=1/tanh(θ)

All the following identities are true for values that do not cause division by zero

Hyperbolic Parity Properties

Sine Odd	sinh(-θ)=-sinh(θ)
Cosine Even	cosh(-θ)=cosh(θ)
Tangent Odd	tanh(-θ)=-tanh(θ)
Cosecant Odd	csch(-θ)=-csch(θ)
Secant Even	sech(-θ)=sech(θ)
Cotangent Odd	coth(-θ)=-coth(θ)

Hyperbolic Pythagorean Identities

Sine-Cosine Pythagorean	$\cosh^2(\theta)-\sinh^2(\theta)=1$
Secant Pythagorean	$1-\tanh^2(\theta)=\text{sech}^2(\theta)$
Cosecant Pythagorean	$\coth^2(\theta)-1=\text{csch}^2(\theta)$

The last two Hyperbolic Pythagorean Identities are obtained by dividing all the terms of the original Sine-Cosine Identity by $\cosh^2(\theta)$ and $\sinh^2(\theta)$, respectively

(H) Half-Angle & Multiple-Angle Identities

Sine Half-Angle	sinh(θ/2)=±√(½(cosh(θ)-1))
Cosine Half-Angle	cosh(θ/2)=√(½(cosh(θ)+1))

(H) Half-Angle & Multiple-Angle Identities (cont)

Tangent Half-Angle 1	tanh(θ/2)=±√((cosh(θ)-1)/(cosh(θ)+1))
Tangent Half-Angle 2	tanh(θ/2)=(cosh(θ)-1)/sinh(θ)
Tangent Half-Angle 3	tanh(θ/2)=sinh(θ)/(cosh(θ)+1)

Sine Double-Angle 1	sinh(2θ)=2sinh(θ)cosh(θ)
Sine Double-Angle 2	$\sinh(2\theta)=2\tanh(\theta)/(1-\tanh^2(\theta))$
Cosine Double-Angle 1	$\cosh(2\theta)=\cosh^2(\theta)+\sinh^2(\theta)$
Cosine Double-Angle 2	$\cosh(2\theta)=2\cosh^2(\theta)-1$
Cosine Double-Angle 3	$\cosh(2\theta)=1+2\sinh^2(\theta)$
Cosine Double-Angle 4	$\cosh(2\theta)=(1+\tanh^2(\theta))/(1-\tanh^2(\theta))$
Tangent Double Angle 1	$\tanh(2\theta)=2\tanh(\theta)/(1+\tanh^2(\theta))$
Tangent Double Angle 2	tanh(2θ)=2/(coth(θ)+tanh(θ))

(H) Half-Angle & Multiple-Angle Identities (cont)

Sine Triple-Angle	$\sinh(3\theta)=3\sinh(\theta)+4\sinh^3(\theta)$
Cosine Triple-Angle	$\cosh(3\theta)=4\cosh^3(\theta)-3\cosh(\theta)$
Tangent Triple-Angle	$\tanh(3\theta)=(3\tan(\theta)+\tan^3(\theta))/(1+3\tan^2(\theta))$

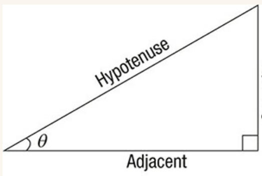
(H) Sum/Difference/Product Identities

Sine Sum/Difference	sinh(θ±φ)	sinh(θ)cosh(φ)±cosh(θ)sinh(φ)
Sine Sum-Product	sinh(θ)±sinh(φ)	2sinh((θ±φ)/2)cosh((θ∓φ)/2)
Sine Product-Sum	sinh(θ)sinh(φ)	½(cosh(θ+φ)-cosh(θ-φ))
Cosine Sum/Difference	cosh(θ±φ)	cosh(θ)cosh(φ)∓sinh(θ)sinh(φ)
Cosine Sum-Product 1	cosh(θ)+cosh(φ)	2cosh((θ+φ)/2)cosh((θ-φ)/2)

(H) Sum/Difference/Product Identities (cont)

Cosine Sum-Product 2	cosh(θ)-cosh(φ)	2sinh((θ+φ)/2)sinh((θ-φ)/2)
Cosine Product-Sum	cosh(θ)cosh(φ)	½(cosh(θ+φ)+cosh(θ-φ))
Sine-Cosine Product-Sum	sinh(θ)cosh(φ)	½(sinh(θ+φ)+sinh(θ-φ))
Tangent Sum/Difference	tanh(θ±φ)	(tanh(θ)±tanh(φ))/(1±tanh(θ)tanh(φ))
Tangent Sum	tanh(θ)±tanh(φ)	sinh(θ±φ)/(cosh(θ)cosh(φ))
Tangent Product	tanh(θ)tanh(φ)	(tanh(θ)+tanh(φ))/(coth(θ)+coth(φ))
Tangent-Cotangent Product	tanh(θ)coth(φ)	(tanh(θ)+coth(φ))/(coth(θ)+tanh(φ))

Right-Triangle Relations



$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Hyperbolic-Inverse Reciprocal Identities

Sine Reciprocal	arcsinh(1/x)=arccsch(x)
Cosine Reciprocal	arccosh(1/x)=arcsech(x)
Tangent Reciprocal	arctanh(1/x)=arccoth(x)
Cosecant Reciprocal	arccsch(1/x)=arcsinh(x)
Secant Reciprocal	arcsech(1/x)=arccosh(x)
Cotangent Reciprocal	arccoth(1/x)=arctanh(x)

(HI) Negative Input Identities

Inverse Sine Odd	arcsinh(-x)=-arcsinh(x)
Inverse Tangent Odd	arctanh(-x)=-arctanh(x)
Inverse Cosecant Odd	arccsch(-x)=-arccsch(x)
Inverse Cotangent Odd	arccoth(-x)=-arccoth(x)

(HI) Half/Multiple Substitution Identities

Half Sine Substitution	½arcsinh(x)=±arcsinh(√(√((1+x^2)-1)/2))
Half Cosine Substitution	½arccosh(x)=arccosh(√((x+1)/2))
Half Tangent Substitution	½arctanh(x)=arctanh(x/(1+√(1-x^2)))
Double Sine Substitution 1	2arcsinh(x)=arcsinh(2x√(1+x^2))
Double Sine Substitution 2	2arcsinh(x)=±arccosh(2x^2+1)
Double Cosine Substitution	2arccosh(x)=arccosh(2x^2-1), *x≥1*
Double Tangent Substitution	2arctanh(x)=arctanh(2x/(1+x^2)), */x/<1*
Triple Sine Substitution	3arcsinh(x)=arcsinh(3x+4x^3)
Triple Cosine Substitution	3arccosh(x)=arccosh(4x^3-3x)
Triple Tangent Substitution	3arctanh(x)=arctanh((3x+x^3)/(1+3x^2))

(HI) Sum/Difference Identities

Sine Sum/Difference	arcsinh(x)±arcsinh(y)=arcsinh(x√(y^2+1)±y√(x^2+1))
Cosine Sum/Difference	arccosh(x)±arccosh(y)=arccosh(xy±√((x^2-1)(y^2-1)))
Sine-Cosine Sum/Difference 1	arcsinh(x)±arccosh(y)=arcsinh(xy±√((x^2+1)(y^2-1))
Sine-Cosine Sum/Difference 2	arcsinh(x)±arccosh(y)=±arccosh(y√(x^2+1)±x√(y^2-1))
Tangent Sum/Difference	arctanh(x)±arctanh(y)=arctanh((x±y)/(1±xy))

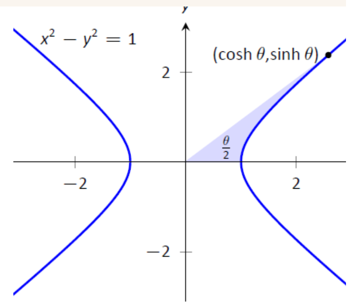
(HI) Logarithmic/Compositional Conversions

Sine Logarithmic	ln(x)=arcsinh((x^2-1)/2x), *x>0*
Cosine Logarithmic	ln(x)=±arccosh((x^2+1)/2x)
Tangent Logarithmic	ln(x)=arctanh((x^2-1)/(x^2+1))
Sine Hyperbolic-Circular 1	arcsinh(tan(x))=ln(sec(x+πn)+tan(x+πn))
Sine Hyperbolic-Circular 2	arcsinh(tan(x))=±arccosh(sec(x+πn))

(HI) Logarithmic/Compositional Conversions (cont)

Tangent Hyperbolic-Circular 1	arctanh(cos(-2x))=ln(|cot(x)|)
Tangent Hyperbolic-Circular 2	±arctanh(sin(x))=±arcsinh(tan(x))
Sine Cofunctional 1	arcsinh(x)=arctanh(x/√(1+x^2))
Sine Cofunctional 2	arcsinh(x)=±arccosh(√(1+x^2))
Cosine Cofunctional 1	arccosh(x)=|arcsinh(√(x^2-1))|, *x≥1*
Cosine Cofunctional 2	arccosh(x)=|arctanh(√(x^2-1)/x)|, *x≥1*
Tangent Cofunctional 1	arctanh(x)=arcsinh(x/√(1-x^2))
Tangent Cofunctional 2	arctanh(x)=±arccosh(1/√(1-x^2))

Unit Hyperbola



x^2-y^2=1