Integral Trigonometry Cheat Sheet

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Basic Trigonometric Integrals

∫sin(x)dx	**-cos(x)+C**
∫cos(x)dx	**sin(x)+C**
∫sec^2(x)dx	**tan(x)+C**
∫sec(x)tan(x)dx	**sec(x)+C**
∫csc^2(x)dx	-cot(x)+C
∫csc(x)cot(x)dx	-csc(x)+C

Common Trigonometric Integrals

∫sin(2x)dx	**-½cos(2x)+C**
∫cos(2x)dx	**½sin(2x)+C** = sin(x)cos(x)+C
∫tan(x)dx	ln|sec(x)|+C
∫sec(x)dx	ln|sec(x)+tan(x)|+C
∫sec^3(x)dx	½(sec(x)tan(x)+ln|sec(x)+tan(x)|)+C
∫csc(x)dx	-ln|csc(x)+cot(x)|+C
∫csc^3(x)dx	-½(csc(x)cot(x)+ln|csc(x)+cot(x)|)+C
∫1/(1+x^2)dx	**arctan(x)+C**
∫1/(a^2+x^2)dx	(1/a)arctan(x/a)+C

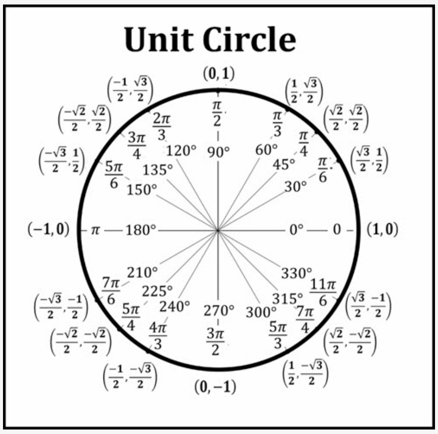
Secant and Cosecant Integral Forms

Secant Integral 1	∫sec(x)dx = ln|sec(x)+tan(x)|+C
Secant Integral 2	∫sec(x)dx = -ln|sec(x)-tan(x)|+C
Secant Integral 3	∫sec(x)dx = ½ln|(sin(x)+1)/(sin(x)-1)|+C
Secant Integral 4	∫sec(x)dx = ln|tan(x/2+π/4)|+C

Secant and Cosecant Integral Forms (cont)

Cosecant Integral 1	∫csc(x)dx = -ln|csc(x)+cot(x)|+C
Cosecant Integral 2	∫csc(x)dx = ln|csc(x)-cot(x)|+C
Cosecant Integral 3	∫csc(x)dx = ½ln|(cos(x)-1)/(cos(x)+1)|+C
Cosecant Integral 4	∫csc(x)dx = ln|tan(x/2)|+C

Sine and Cosine Unit Circle



Powers of Trigonometric Functions

https://cheatography.com/crossant/cheat-sheets/integral-cases-for-trigonometric-p-owers/

Quotient and Reciprocal Identities

Tangent Quotient	**tan(x)=sin(x)/cos(x)**
Cotangent Quotient	**cot(x)=cos(x)/sin(x)**
Sine Reciprocal	sin(x)=1/csc(x)
Cosine Reciprocal	cos(x)=1/sec(x)
Tangent Reciprocal	tan(x)=1/cot(x)
Cosecant Reciprocal	**csc(x)=1/sin(x)**
Secant Reciprocal	**sec(x)=1/cos(x)**
Cotangent Reciprocal	cot(x)=1/tan(x)

Pythagorean Identities

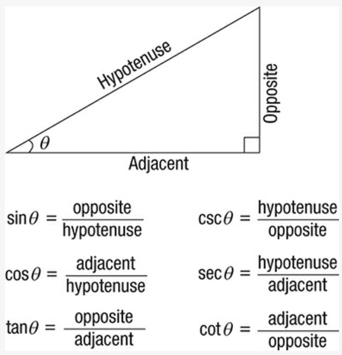
Sine-Cosine Pythagorean	**sin^2(x)+cos^2(x)=1**
Sine Pythagorean	**sin^2(x)=1-cos^2(x)**
Cosine Pythagorean	**cos^2(x)=1-sin^2(x)**
Secant Pythagorean	**tan^2(x)+1=sec^2(x)**
Tangent Pythagorean	**tan^2(x)=sec^2(x)-1**
Secant-Tangent Pythagorean	sec^2(x)-tan^2(x)=1
Cosecant Pythagorean	1+cot^2(x)=csc^2(x)
Cotangent Pythagorean	cot^2(x)=csc^2(x)-1
Cosecant-Cotangent Pythagorean	csc^2(x)-cot^2(x)=1

The last two triplets of Pythagorean identities are obtained by dividing all the terms of the original identity by sin²(x) or cos²(x)

Sum, Difference, and Product Identities

sin(x+y)=sin(x)cos(y) + cos(x)sin(y)
sin(x-y)=sin(x)cos(y) - cos(x)sin(y)
cos(x+y)=cos(x)cos(y) - sin(x)sin(y)
cos(x-y)=cos(x)cos(y) + sin(x)sin(y)
sin(x)sin(y)=½(cos(x-y) - cos(x+y))
cos(x)cos(y)=½(cos(x+y) + cos(x-y))
sin(x)cosy)=½(sin(x+y) + sin(x-y))

Right-Triangle Trigonometric Relations



Trigonometric Substitutions

a^2-u^2	**Let u=asin(θ), du=acos(θ)dθ**	θ∈[-π/2, π/2]
u^2-a^2	**Let u=asec(θ), du=asec(θ)tan(θ)dθ**	θ∈[0, π/2)
u^2+a^2	**Let u=atan(θ), du=asec^2(θ)dθ**	θ∈(-π/2, π/2)
$a^2-b^2u^2$	Let u=(a/b)sin(θ), du=(a/b)cos(θ)dθ	θ∈[-π/2, π/2]
$b^2u^2-a^2$	Let u=(a/b)sec(θ), du=(a/b)sec(θ)tan(θ)dθ	θ∈[0, π/2)
$b^2u^2+a^2$	Let u=(a/b)tan(θ), du=(a/b)sec^2(θ)dθ	θ∈(-π/2, π/2)

Trigonometric substitutions are typically used under radicals, however, they are not required to be

For definite integrals, you will need to set u equal to its respective bounds, and solve for θ in order to properly change the bounds of integration with respect to θ

Half-Angle and Double-Angle Identities

Sine Half-Angle	sin(x/2)=√(½(1-cos(x)))
Cosine Half-Angle	cos(x/2)=√(½(1+cos(x)))
Sine Power-Reducing	**sin^2(x)=½(1-cos(2x))**
Cosine Power-Reducing	**cos^2(x)=½(1+cos(2x))**
Sine Double-Angle	**sin(2x)=2sin(x)cos(x)**
Cosine Double-Angle 1	**cos(2x)=cos^2(x)-sin^2(x)**
Cosine Double-Angle 2	cos(2x)=2cos^2(x)-1

Half-Angle and Double-Angle Identities (cont)

Cosine Double-Angle 3	cos(2x)=1-2sin^2(x)

Sine Power-Reducing and Cosine Power-Reducing identities are variations of the Half-Angle identities

Tangent Unit Circle