

### Basic Trigonometric Integrals

$\int \sin(x) dx$	$-\cos(x)+C$
$\int \cos(x) dx$	$\sin(x)+C$
$\int \sec^2(x) dx$	$\tan(x)+C$
$\int \sec(x)\tan(x) dx$	$\sec(x)+C$
$\int \csc^2(x) dx$	$-\cot(x)+C$
$\int \csc(x)\cot(x) dx$	$-\csc(x)+C$

### Common Trigonometric Integrals

$\int \sin(2x) dx$	$-\frac{1}{2}\cos(2x)+C$
$\int \cos(2x) dx$	$\frac{1}{2}\sin(2x)+C = \sin(x)\cos(x)+C$
$\int \tan(x) dx$	$\ln \sec(x) +C$
$\int \sec(x) dx$	$\ln \sec(x)+\tan(x) +C$
$\int \sec^3(x) dx$	$\frac{1}{2}(\sec(x)\tan(x)+\ln \sec(x)+\tan(x) )+C$
$\int \csc(x) dx$	$-\ln \csc(x)+\cot(x) +C$
$\int \csc^3(x) dx$	$-\frac{1}{2}(\csc(x)\cot(x)+\ln \csc(x)+\cot(x) )+C$
$\int 1/(1+x^2) dx$	$\arctan(x)+C$
$\int 1/(a^2+x^2) dx$	$(1/a)\arctan(x/a)+C$

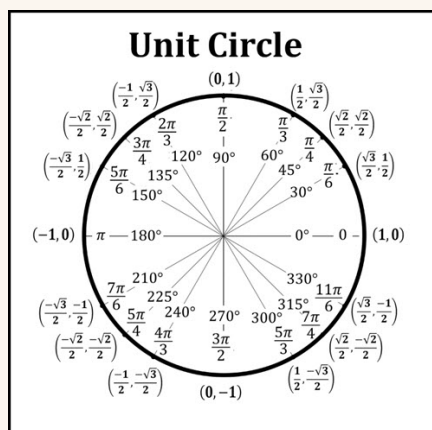
### Secant and Cosecant Integral Forms

Secant Integral 1	$\int \sec(x) dx = \ln \sec(x)+\tan(x) +C$
Secant Integral 2	$\int \sec(x) dx = -\ln \sec(x)-\tan(x) +C$
Secant Integral 3	$\int \sec(x) dx = \frac{1}{2}\ln (\sin(x)+1)/(\sin(x)-1) +C$
Secant Integral 4	$\int \sec(x) dx = \ln \tan(x/2+2+\pi/4) +C$

### Secant and Cosecant Integral Forms (cont)

Cosecant Integral 1	$\int \csc(x) dx = -\ln \csc(x)+\cot(x) +C$
Cosecant Integral 2	$\int \csc(x) dx = \ln \csc(x)-\cot(x) +C$
Cosecant Integral 3	$\int \csc(x) dx = \frac{1}{2}\ln (\cos(x)-1)/(\cos(x)+1) +C$
Cosecant Integral 4	$\int \csc(x) dx = \ln \tan(x/2) +C$

### Sine and Cosine Unit Circle



### Powers of Trigonometric Functions

<https://cheatography.com/crossant/cheat-sheets/integral-cases-for-trigonometric-powers/>

### Quotient and Reciprocal Identities

Tangent Quotient	$\tan(x)=\sin(x)/\cos(x)$
Cotangent Quotient	$\cot(x)=\cos(x)/\sin(x)$
Sine Reciprocal	$\sin(x)=1/\csc(x)$
Cosine Reciprocal	$\cos(x)=1/\sec(x)$
Tangent Reciprocal	$\tan(x)=1/\cot(x)$
Cosecant Reciprocal	$\csc(x)=1/\sin(x)$
Secant Reciprocal	$\sec(x)=1/\cos(x)$
Cotangent Reciprocal	$\cot(x)=1/\tan(x)$

### Pythagorean Identities

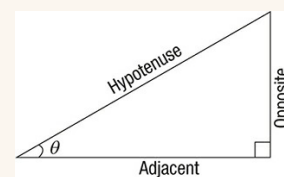
Sine-Cosine Pythagorean	$\sin^2(x)+\cos^2(x)=1$
Sine Pythagorean	$\sin^2(x)=1-\cos^2(x)$
Cosine Pythagorean	$\cos^2(x)=1-\sin^2(x)$
Secant Pythagorean	$\tan^2(x)+1=\sec^2(x)$
Tangent Pythagorean	$\tan^2(x)=\sec^2(x)-1$
Secant-Tangent Pythagorean	$\sec^2(x)-\tan^2(x)=1$
Cosecant Pythagorean	$1+\cot^2(x)=\csc^2(x)$
Cotangent Pythagorean	$\cot^2(x)=\csc^2(x)-1$
Cosecant-Cotangent Pythagorean	$\csc^2(x)-\cot^2(x)=1$

The last two triplets of Pythagorean identities are obtained by dividing all the terms of the original identity by  $\sin^2(x)$  or  $\cos^2(x)$

### Sum, Difference, and Product Identities

$\sin(x+y)=\sin(x)\cos(y) + \cos(x)\sin(y)$
$\sin(x-y)=\sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y) - \sin(x)\sin(y)$
$\cos(x-y)=\cos(x)\cos(y) + \sin(x)\sin(y)$
$\sin(x)\sin(y)=\frac{1}{2}(\cos(x-y) - \cos(x+y))$
$\cos(x)\cos(y)=\frac{1}{2}(\cos(x+y) + \cos(x-y))$
$\sin(x)\cos(y)=\frac{1}{2}(\sin(x+y) + \sin(x-y))$

### Right-Triangle Trigonometric Relations



$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$	$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$

### Trigonometric Substitutions

$a^2-u^2$	Let $u=asin(\theta)$ , $du=acos(\theta)d\theta$	$\theta \in [-\pi/2, \pi/2]$
$u^2-a^2$	Let $u=asec(\theta)$ , $du=asec(\theta)\tan(\theta)d\theta$	$\theta \in [0, \pi/2)$
$u^2+a^2$	Let $u=atan(\theta)$ , $du=asec^2(\theta)d\theta$	$\theta \in (-\pi/2, \pi/2)$
$a^2-b^2u^2$	Let $u=(a/b)\sin(\theta)$ , $du=(a/b)\cos(\theta)d\theta$	$\theta \in [-\pi/2, \pi/2]$
$b^2u^2-a^2$	Let $u=(a/b)\sec(\theta)$ , $du=(a/b)\sec(\theta)\tan(\theta)d\theta$	$\theta \in [0, \pi/2)$
$b^2u^2+a^2$	Let $u=(a/b)\tan(\theta)$ , $du=(a/b)\sec^2(\theta)d\theta$	$\theta \in (-\pi/2, \pi/2)$

Trigonometric substitutions are typically used under radicals, however, they are not required to be

For definite integrals, you will need to set  $u$  equal to its respective bounds, and solve for  $\theta$  in order to properly change the bounds of integration with respect to  $\theta$

### Half-Angle and Double-Angle Identities

Sine Half-Angle	$\sin(x/2) = \sqrt{\frac{1-\cos(x)}{2}}$
Cosine Half-Angle	$\cos(x/2) = \sqrt{\frac{1+\cos(x)}{2}}$
Sine Power-Reducing	$\sin^2(x) = \frac{1-\cos(2x)}{2}$
Cosine Power-Reducing	$\cos^2(x) = \frac{1+\cos(2x)}{2}$
Sine Double-Angle	$\sin(2x) = 2\sin(x)\cos(x)$
Cosine Double-Angle 1	$\cos(2x) = \cos^2(x) - \sin^2(x)$
Cosine Double-Angle 2	$\cos(2x) = 2\cos^2(x) - 1$

### Half-Angle and Double-Angle Identities (cont)

Cosine Double-Angle  $\cos(2x) = 1 - 2\sin^2(x)$

Sine Power-Reducing and Cosine Power-Reducing identities are variations of the Half-Angle identities

### Tangent Unit Circle

