Integral Trigonometry Cheat Sheet

by CROSSANT (CROSSANT) via cheatography.com/186482/cs/38992/

Basic Trigonometric Integrals

$\int \sin(x)dx$	**-cos(x)+C**
$\int \cos(x)dx$	**sin(x)+C**
$\int \sec^2(x)dx$	**tan(x)+C**
$\int \sec(x)\tan(x)dx$	**sec(x)+C**
$\int \csc^2(x)dx$	-cot(x)+C
$\int \csc(x)\cot(x)dx$	-csc(x)+C

Common Trigonometric Integrals

$\int \sin(2x)dx$	**-½cos(2x)+C**
$\int \cos(2x)dx$	**½sin(2x)+C** = sin(x)cos(x)+C
$\int \tan(x)dx$	ln|sec(x)|+C
$\int \sec(x)dx$	ln|sec(x)+tan(x)|+C
$\int \sec^3(x)dx$	½(sec(x)tan(x)+ln|sec(x)+tan(x)|)+C
$\int \csc(x)dx$	-ln|csc(x)+cot(x)|+C
$\int \csc^3(x)dx$	-½(csc(x)cot(x)+ln|csc(x)+cot(x)|)+C
$\int 1/(1+x^2)dx$	**arctan(x)+C**
$\int 1/(a^2+x^2)dx$	(1/a)arctan(x/a)+C

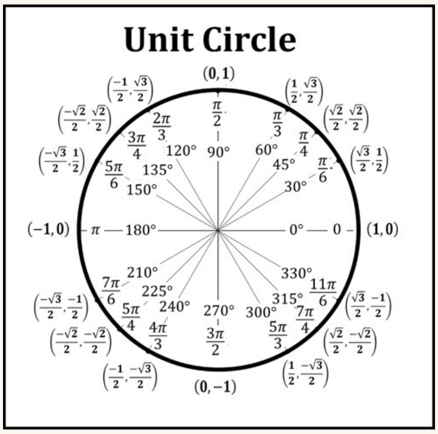
Secant and Cosecant Integral Forms

Secant Integral 1	$\int$sec(x)dx = ln|sec(x)+tan(x)|+C
Secant Integral 2	$\int$sec(x)dx = -ln|sec(x)-tan(x)|+C
Secant Integral 3	$\int$sec(x)dx = ½ln|(sin(x)+1)/(sin(x)-1)|+C
Secant Integral 4	$\int$sec(x)dx = ln|tan(x/2+π/4)|+C

Secant and Cosecant Integral Forms (cont)

Cosecant Integral 1	$\int$csc(x)dx = -ln|csc(x)+cot(x)|+C
Cosecant Integral 2	$\int$csc(x)dx = ln|csc(x)-cot(x)|+C
Cosecant Integral 3	$\int$csc(x)dx = ½ln|(cos(x)-1)/(cos(x)+1)|+C
Cosecant Integral 4	$\int$csc(x)dx = ln|tan(x/2)|+C

Sine and Cosine Unit Circle



Powers of Trigonometric Functions

https://cheatography.com/crossant/cheat-sheets/integral-cases-for-trigonometric-powers/

Quotient and Reciprocal Identities

Tangent Quotient	**tan(x)=sin(x)/cos(x)**
Cotangent Quotient	**cot(x)=cos(x)/sin(x)**
Sine Reciprocal	sin(x)=1/csc(x)
Cosine Reciprocal	cos(x)=1/sec(x)
Tangent Reciprocal	tan(x)=1/cot(x)
Cosecant Reciprocal	**csc(x)=1/sin(x)**
Secant Reciprocal	**sec(x)=1/cos(x)**
Cotangent Reciprocal	cot(x)=1/tan(x)

Pythagorean Identities

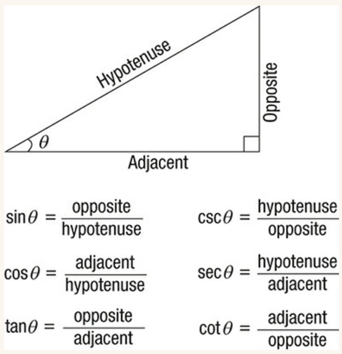
Sine-Cosine Pythagorean	**$\sin^2(x)+\cos^2(x)=1$**
Sine Pythagorean	**$\sin^2(x)=1-\cos^2(x)$**
Cosine Pythagorean	**$\cos^2(x)=1-\sin^2(x)$**
Secant Pythagorean	**$\tan^2(x)+1=\sec^2(x)$**
Tangent Pythagorean	**$\tan^2(x)=\sec^2(x)-1$**
Secant-Tangent Pythagorean	$\sec^2(x)-\tan^2(x)=1$
Cosecant Pythagorean	$1+\cot^2(x)=\csc^2(x)$
Cotangent Pythagorean	$\cot^2(x)=\csc^2(x)-1$
Cosecant-Cotangent Pythagorean	$\csc^2(x)-\cot^2(x)=1$

The last two triplets of Pythagorean identities are obtained by dividing all the terms of the original identity by $\sin^2(x)$ or $\cos^2(x)$

Sum, Difference, and Product Identities

sin(x+y)=sin(x)cos(y) + cos(x)sin(y)
sin(x-y)=sin(x)cos(y) - cos(x)sin(y)
cos(x+y)=cos(x)cos(y) - sin(x)sin(y)
cos(x-y)=cos(x)cos(y) + sin(x)sin(y)
sin(x)sin(y)=½(cos(x-y) - cos(x+y))
cos(x)cos(y)=½(cos(x+y) + cos(x-y))
sin(x)cosy)=½(sin(x+y) + sin(x-y))

Right-Triangle Trigonometric Relations



Trigonometric Substitutions

a^2-u^2	**Let u=asin(θ), du=acos(θ)dθ**	θ∈[-π/2, π/2]
u^2-a^2	**Let u=asec(θ), du=asec(θ)tan(θ)dθ**	θ∈[0, π/2)
u^2+a^2	**Let u=atan(θ), du=asec^2(θ)dθ**	θ∈(-π/2, π/2)
a^2-b^2u^2	Let u=(a/b)sin(θ), du=(a/b)cos(θ)dθ	θ∈[-π/2, π/2]
b^2u^2-a^2	Let u=(a/b)sec(θ), du=(a/b)sec(θ)tan(θ)dθ	θ∈[0, π/2)
b^2u^2+a^2	Let u=(a/b)tan(θ), du=(a/b)sec^2(θ)dθ	θ∈(-π/2, π/2)

Trigonometric substitutions are typically used under radicals, however, they are not required to be

For definite integrals, you will need to set u equal to its respective bounds, and solve for θ in order to properly change the bounds of integration with respect to θ

Half-Angle and Double-Angle Identities

Sine Half-Angle	sin(x/2)=√(½(1-cos(x)))
Cosine Half-Angle	cos(x/2)=√(½(1+cos(x)))
Sine Power-Reducing	**sin^2(x)=½(1-cos(2x))**
Cosine Power-Reducing	**cos^2(x)=½(1+cos(2x))**
Sine Double-Angle	**sin(2x)=2sin(x)cos(x)**
Cosine Double-Angle 1	**cos(2x)=cos^2(x)-sin^2(x)**
Cosine Double-Angle 2	cos(2x)=2cos^2(x)-1

Half-Angle and Double-Angle Identities (cont)

Cosine Double-Angle 3	cos(2x)=1-2sin^2(x)

Sine Power-Reducing and Cosine Power-Reducing identities are variations of the Half-Angle identities

Tangent Unit Circle