

Basic Trigonometric Integrals

$\int \sin(x) dx$	$-\cos(x)+C$
$\int \cos(x) dx$	$\sin(x)+C$
$\int \sec^2(x) dx$	$\tan(x)+C$
$\int \sec(x)\tan(x) dx$	$\sec(x)+C$
$\int \csc^2(x) dx$	$-\cot(x)+C$
$\int \csc(x)\cot(x) dx$	$-\csc(x)+C$

Common Trigonometric Integrals

$\int \sin(2x) dx$	$-\frac{1}{2}\cos(2x)+C$
$\int \cos(2x) dx$	$\frac{1}{2}\sin(2x)+C = \sin(x)\cos(x)+C$
$\int \tan(x) dx$	$\ln \sec(x) +C$
$\int \sec(x) dx$	$\ln \sec(x)+\tan(x) +C$
$\int \sec^3(x) dx$	$\frac{1}{2}(\sec(x)\tan(x)+\ln \sec(x)+\tan(x))+C$
$\int \csc(x) dx$	$-\ln \csc(x)+\cot(x) +C$
$\int \csc^3(x) dx$	$-\frac{1}{2}(\csc(x)\cot(x)+\ln \csc(x)-\cot(x))+C$
$\int 1/(1+x^2) dx$	$\arctan(x)+C$
$\int 1/(a^2+x^2) dx$	$(1/a)\arctan(x/a)+C$

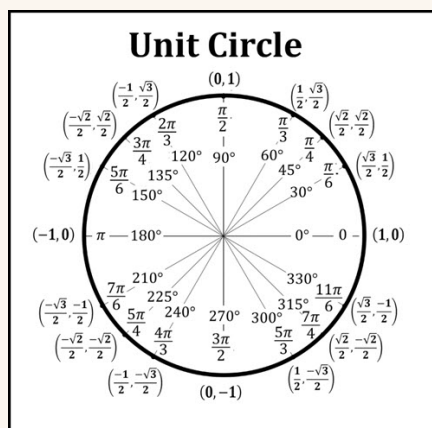
Secant and Cosecant Integral Forms

Secant Integral 1	$\int \sec(x) dx = \ln \sec(x)+\tan(x) +C$
Secant Integral 2	$\int \sec(x) dx = -\ln \sec(x)-\tan(x) +C$
Secant Integral 3	$\int \sec(x) dx = \frac{1}{2}\ln (\sin(x)+1)/(\sin(x)-1) +C$
Secant Integral 4	$\int \sec(x) dx = \ln \tan(x)/2+\pi/4 +C$

Secant and Cosecant Integral Forms (cont)

Cosecant Integral 1	$\int \csc(x) dx = \ln \csc(x)-\cot(x) +C$
Cosecant Integral 2	$\int \csc(x) dx = -\ln \csc(x)+\cot(x) +C$
Cosecant Integral 3	$\int \csc(x) dx = \frac{1}{2}\ln (\cos(x)-1)/(\cos(x)+1) +C$
Cosecant Integral 4	$\int \csc(x) dx = \ln \tan(x/2) +C$

Sine and Cosine Unit Circle



Powers of Trigonometric Functions

<https://cheatography.com/crossant/cheat-sheets/integral-cases-for-trigonometric-powers/>

Quotient and Reciprocal Identities

Tangent Quotient	$\tan(x)=\sin(x)/\cos(x)$
Cotangent Quotient	$\cot(x)=\cos(x)/\sin(x)$

Sine Reciprocal $\sin(x)=1/\csc(x)$

Cosine Reciprocal $\cos(x)=1/\sec(x)$

Tangent Reciprocal $\tan(x)=1/\cot(x)$

Cosecant Reciprocal $\csc(x)=1/\sin(x)$

Secant Reciprocal $\sec(x)=1/\cos(x)$

Cotangent Reciprocal $\cot(x)=1/\tan(x)$

Pythagorean Identities

Sine-Cosine Pythagorean	$\sin^2(x)+\cos^2(x)=1$
Sine Pythagorean	$\sin^2(x)=1-\cos^2(x)$
Cosine Pythagorean	$\cos^2(x)=1-\sin^2(x)$
Secant Pythagorean	$\tan^2(x)+1=\sec^2(x)$
Tangent Pythagorean	$\tan^2(x)=\sec^2(x)-1$
Secant-Tangent Pythagorean	$\sec^2(x)-\tan^2(x)=1$
Cosecant Pythagorean	$1+\cot^2(x)=\csc^2(x)$
Cotangent Pythagorean	$\cot^2(x)=\csc^2(x)-1$
Cosecant-Cotangent Pythagorean	$\csc^2(x)-\cot^2(x)=1$

The last two triplets of Pythagorean identities are obtained by dividing all the terms of the original identity by $\sin^2(x)$ or $\cos^2(x)$

Sum and Difference Identities

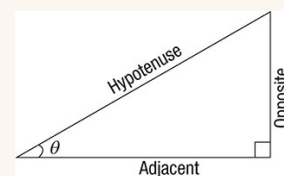
$\sin(x+y)=\sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x-y)=\sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x-y)=\cos(x)\cos(y) + \sin(x)\sin(y)$

Right-Triangle Trigonometric Relations



$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ $\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$



Trigonometric Substitutions

a^2-x^2	Let $x=asin(\theta)$	$dx=acos(\theta)d\theta$
x^2-a^2	Let $x=asec(\theta)$	$dx=asec(\theta)tan(\theta)d\theta$
x^2+a^2	Let $x=atan(\theta)$	$dx=asec^2(\theta)d\theta$
$a^2-b^2x^2$	Let $x=(a/b)sin(\theta)$	$dx=(a/b)cos(\theta)d\theta$
$b^2x^2-a^2$	Let $x=(a/b)sec(\theta)$	$dx=(a/b)sec(\theta)tan(\theta)d\theta$
$b^2x^2+a^2$	Let $x=(a/b)tan(\theta)$	$dx=(a/b)sec^2(\theta)d\theta$

Trigonometric substitutions are typically used under radicals, however, they are not required to be

For definite integrals, you will need to set x equal to its respective bounds, and solve for θ in order to properly change the bounds of integration with respect to θ

Half-Angle and Double-Angle Identities

Sine Half-Angle	$sin(x/2)=\sqrt{1/2(1-cos(x))}$
Cosine Half-Angle	$cos(x/2)=\sqrt{1/2(1+cos(x))}$
Sine Power-Reducing	$sin^2(x)=1/2(1-cos(2x))$
Cosine Power-Reducing	$cos^2(x)=1/2(1+cos(2x))$
Sine Double-Angle	$sin(2x)=2sin(x)cos(x)$
Cosine Double-Angle 1	$cos(2x)=cos^2(x)-sin^2(x)$
Cosine Double-Angle 2	$cos(2x)=2cos^2(x)-1$
Cosine Double-Angle 3	$cos(2x)=1-2sin^2(x)$

Sine Power-Reducing and Cosine Power-Reducing identities are variations of the Half-Angle identities

Tangent Unit Circle

