

## Calculus II Cheat Sheet by CROSSANT (CROSSANT) via cheatography.com/186482/cs/38975/

Series				
Series Type	General Summation	Convergence	Divergence	Notes
Infinite Series	$\Sigma a_n = \Sigma^{\infty}_{n=k} a_n$ , where k is some whole number (k = {0, 1, 2,}) for which $a_n$ is well-defined	Varies	Varies	
Harmonic Series	Σ1/n	Never converges	Always diverges	The alternating version of this series $(\Sigma(-1)^{n+1}/n)$ converges, and $\Sigma 1/n$ is a P-Series with p=1
Geometric Series	$\Sigma^{\infty}_{n=0}$ ar $^{n}=\Sigma^{\infty}_{n=1}$ ar $^{n-1}$ , where a is the first term of the series, and r is the common ratio between terms	Converges if  r <1	Diverges if  r ≥1	If the series converges, its sum is S=a/(1-r)
P-Series	$\Sigma 1/n^p$ , where p is a positive number	Converges if p>1	Diverges if p	≤1
Altern- ating Series	$\Sigma(-1)^{n}b_{n}$ , $\Sigma(-1)^{n+1}b_{n}$ , or $\Sigma(-1)^{n-1}b_{n}$	Converges if $\lim_{n\to\infty} b_n = 0$ and $b_n$ is a decreasing sequence (or eventually decreasing)	Cannot show	v divergence, inconclusive
Telesc- oping Series	$\Sigma(b_n-b_{n+1})$	Varies	Varies	

Alternating Series Estimation Theorem: If  $S_n = \Sigma^n_{i=1}(-1)^i b_i$  or  $\Sigma^n_{i=1}(-1)^{i-1} b_i$  is the sum of an alternating series that converges, then  $|R_n| = |S - S_n| \le b_{n+1}$ 

Trigonometric functions like  $cos(n\pi)$  or  $sin(n\pi+\pi/2)$  act as sign alternators, like  $(-1)^n$ 

The Alternating Series Test (AST) checks the limit, but since the AST only concludes convergence, we explicitly apply the Test For Divergence (albeit, redundantly) if the limit fails (even though it checks the same limit)

Series Tests				
Test Type	Typical series to use test	Convergence	Divergence	Notes
Test for Divergence	Σa <sub>n</sub>	Cannot show convergence, inconclusive	Diverges if lim	n->∞ a <sub>n</sub> ≠0 or DNE
(Direct) Comparison Test	$a_n$ and $b_n$ are positive-termed ( $a_n \ge 0$ ) and $b_n \ge 0$ for all n) and $a_n \le b_n$ for all n	$\Sigma a_n$ converges if $\Sigma b_n$ converges	$\Sigma b_n$ diverges if $\Sigma a_n$ diverges	Inconclusive if $\mathbf{b_n}$ diverges in proving convergence, or $\mathbf{a_n}$ converges in proving divergence



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Published 28th May, 2023. Last updated 20th October, 2025. Page 1 of 5.



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Series Tests (cont)				
Limit Comparison Test	$a_{n}$ and $b_{n}$ are positive-termed, and lim $_{n>\infty}$ $a_{n}/b_{n}\text{=-}c,$ where $0\text{<-}c\text{<-}\infty$	$\Sigma a_n$ converges $\iff$ $\Sigma b_n$ converges	$\Sigma a_n$ diverges $\Longleftrightarrow$ $\Sigma b_n$ diverges	Inconclusive if c=0, c= $\infty$ , or c DNE
Ratio Test	$\Sigma a_n$ , usually with n! terms, product terms, or $(a_n)^n$	Absolutely converges if $\lim_{n\to\infty}  a_{n+1}/a_n  < 1$	Diverges if $\lim_{n\to\infty}  a_{n+1}/a_n  > 1$	Inconclusive if $\lim_{n\to\infty}$ $ a_{n+1}/a_n =1$
Root Test	$Σa_n$ , usually with $(a_n)^n$	Absolutely converges if $\lim_{n\to\infty}  a_n ^{1/n} < 1$	Diverges if $\lim_{n\to\infty}  a_n ^{1/n} > 1$	Inconclusive if $\lim_{n\to\infty}$ $ a_n ^{1/n}=1$
Integral Test	$a_n{\equiv}f(x), \text{ which is a positive, continuous, decreasing}$ function on the interval $[k,{}^{_{\!$	Converges if $\int_{\mathbf{k}}^{\infty}$ f(x)dx converges	Diverges if ∫ <sub>k</sub> °	<sup>∞</sup> f(x)dx diverges
Absolute/Conditional Convergence Classification	Σa <sub>n</sub>	Absolutely converges if $\Sigma  a_n $ converges	Diverges if Σa <sub>n</sub> diverges	Conditionally converges if $\Sigma  a_n  \text{ diverges, but } \Sigma a_n$ converges

For the series listed, assume each series to be an infinite series starting at n=k:  $\Sigma^{\circ}_{n=k}$ = $\Sigma$  as previously defined

If a test is inconclusive, use another test

The symbol [  $\iff$  ] represents the relationship "if and only if" (often abbreviated to "iff"), meaning both sides of the statement must be true at the same time, or false at the same time

Special Series			
Series	Summation Form	First five terms	Radius & Interval of Convergence
Power Series centered at a	$\Sigma C_n(x-a)^n$	$C_0+C_1(x-a)+C_2(x-a)^2+C_3(x-a)^3+C_4(x-a)^4+$	(a-R, a+R), [a-R, a+R), (a-R, a+R], or [a-R, a+R]
Taylor Series centered at a	$\Sigma f^{(n)}(a)(x-a)^n/n!$	$f(a)+f'(a)(x-a)+f''(a)(x-a)^{2}/2!+f'''(a)(x-a)^{3}/3!+f^{(4)}(a)(x-a)^{4}/4!+$	x-a  <r< td=""></r<>
Maclaurin Series (Taylor Series centered at 0)	$\Sigma f^{(n)}(0)(x-0)^n/n! = \Sigma f^{(n)}(0)x^n/n!$	f(0)+f'(0)x+- $f''(0)x^2/2!+f'''(0)x^3/3!+f^{(4)}(0)x^4/4!+$	x  <r< td=""></r<>
1/(1-x)	$\Sigma x^n$	$1+x+x^2+x^3+x^4+$	R=1, I=(-1,1)
e <sup>X</sup>	$\Sigma x^{n}/n!$	$1+x+x^2/2!+x^3/3!+x^4/4!+$	R=∞, I=(-∞,∞)
In(1+x)	$\Sigma(-1)^{n}x^{n+1}/(n+1)$	$x-x^2/2+x^3/3-x^4/4+x^5/5$	R=1, I=(-1,1]
arctan(x)	$\Sigma(-1)^n x^{2n+1}/(2n+1)$	$x-x^3/3+x^5/5-x^7/7+x^9/9$	R=1, I=[-1,1]



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Published 28th May, 2023. Last updated 20th October, 2025. Page 2 of 5.



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#### Special Series (cont) $\Sigma(-1)^n x^{2n+1}/(2n+1)!$ $x-x^3/3!+x^5/5!-x^7/7!+x^9/9!-...$ sin(x) $R=\infty$ , $I=(-\infty,\infty)$ $1-x^2/2!+x^4/4!-x^6/6!+x^8/8!-...$ $\Sigma(-1)^n x^{2n}/(2n)!$ $R=\infty$ , $I=(-\infty,\infty)$ cos(x) $(1+x)^{k}$ $\Sigma(_{n}^{k})x^{n}=\Sigma((k(k-1)(k-2)(k-3)...(k-n+1))/n!)x^{n}$ $1+kx+k(k-1)x^2/2!+k(k-1)(k-2)x^3/3!+k(k-1)(k-2)(k-3)x^4/4!+...$ R=1 $|R_n(x)| \le M|x-a|^{n+1}/(n+1)!$ , where M is a constant such that $|f^{(n+1)}(x)| \le M$ for all $|x-a| \le d$ Taylor's Inequality

For the series listed, assume each series to be an infinite series starting at n=0:  $\Sigma^{\circ}_{n=0}$ = $\Sigma$  as previously defined

The Radius of Convergence, R, is typically found by using the Ratio Test or Root Test

 $\binom{k}{n}$  is the "binomial coefficient" (read as "k choose n").  $\binom{k}{n} = \frac{k!}{(n!(k-n)!)}$ 

f<sup>(n)</sup> means "the nth derivative of the function f"

n!=n(n-1)!=n(n-1)(n-2)!=n(n-1)(n-2)(n-3)!=...

n! = n(n-1)(n-2)(n-3)...\*3\*2\*1

0!=1, 1!=1

Areas of Fu	unctions	
Between two functions	$\int_{a}^{b}$ ((top function)- (bottom function))dA	
Enclosed by a polar function	$\frac{1}{2}\int_{a}^{b}f(\theta)^{2}d\theta$	
Between two polar functions	$\frac{1}{2}\int_{a}^{b}$ ((outer polar function) <sup>2</sup> -(inner polar function) <sup>2</sup> )d $\theta$	
Area enclosed by a polar function is with respect to the pole, which is the origin  Average value of a function on an interval: $f_{avg} = 1/(b-a) \int_{a}^{b}$		
f(x)dx		

#### Volumes of Solids of Revolution

Disk	π∫a <sup>b</sup> (radius) <sup>2</sup> dr
Washer	$\pi \int_a^b ((outer radius)^2)$
	- (inner radius)) <sup>2</sup> dr
Cylind-	2π∫a <sup>b</sup> (radius)(hei-
rical	ght)dr
Shell	

Disk: slices are perpendicular and connected to axis of rotation Washer: slices are perpendicular and disconnected to axis of rotation

Cylindrical Shells: slices are parallel to axis of rotation

Arc Lengths

Surface Areas	
Function revolved about an axis	$2\pi\int_{a}^{b}$ (radius)(Arc Length component)ds
Function revolved about y-axis	$2\pi \int_{a}^{b} x\sqrt{1+}$ $(f'(x))^{2})dx$
Function revolved about x-axis	$2\pi \int_{a}^{b} y\sqrt{1+}$ $(g'(y))^{2})dy$
Parametric function of t revolved about y-axis	$2\pi \int_{a}^{b} f(x)\sqrt{((x'(t))^{2}+} (y'(t))^{2})dt$
Parametric function of t revolved about x-axis	$2\pi \int_{a}^{b}$ $g(y)\sqrt{((x'(t))^{2}+}$ $(y'(t))^{2})dt$

f'(x)=dy/dx, g'(y)=dx/dy, x'(t)=dx/dt, and y'(t)=dy/dt

integration by Parts		
Indefinite	∫udv=uv-∫vdu	
Integral		
Definite	$\int_a^b udv = uv _a^b$ -	
Integral	$\int_{\mathbf{a}}^{\mathbf{b}} vdu$	

Integration by Parts is used to integrate integrals that have components multiplied together in their simplest form, often referred to as a "product rule for integrals"

Choosing the "dv" term depends

Trigonometric Integrals
https://cheatography.com/cross-
ant/cheat-sheets/integral-trig-
onometry/

Integration by Partial Fractions

(ax+b)/(px+q)(rx+s)	A/(px+g) at least one discontinuity on B/(rx+g)he integrated region (Type 2)
$(ax+b)/(px+q)^2$	A/(px+q) + B/(px Cenic Sections
$(ax^2+bx+c)/((px+q)$ $(rx^2+sx+t))$	A/(px+q)trbs://cheatography.com/cross- (Bx+C)e(nx/c2eat-sheets/conic-sections/
$(ax^3+bx^2+cx+d)/(rx^2+sx+t)^2$	+sx+t) (Ax+E Parametric Curves and Polar

Improper Integrals (cont)

lim t->±∞

Improper Integrals are integrals with bounds at infinity (Type 1)

 $f(x)dx=\pm\infty$  or DNE

Divergence

of  $\int f(x)dx$ 

**Functions**  $(Cx+D)/(rx^2+sx+t)^2$ 

For numerator/denominator N/D, partial fractions simplify integrals of polynomial rational expressions where deg(N) < deg(D), and a fully-factored D, into a sum of simpler fractions. If  $deg(N) \ge deg(D)$ , then polynomial long division must be used beforehand.

A non-repeated denominator factor of degree n gets a partial fraction with numerator degree n-1.

An irreducible denominator factor repeated p times (like (ax+b)^p) of degree n gets p partial fractions, all with numerator-degree n-1.

Function	∫a <sup>b</sup> √(1+
	$(f'(x))^2)dx$
Parametric	$\int_a{}^b\sqrt{((x'(t))^2}+$
Function	$(y'(t))^2)dt$
Polar	$\int_a^b \sqrt{(r(\theta)^2 + }$
Function	$(r'(\theta))^2)d\theta$

For standard functions: f'(x)=dy/dx For parametric functions: x'(t)=-

dx/dt and y'(t)=dy/dtFor polar functions:  $r'(\theta)=dr/d\theta$  on what will simplify the integral the best, while being relatively simple to integrate

The constant of integration does not need to be inserted until the integral has been fully simplified

Improper Integrals		
$\int_{a}^{\infty} f(x) dx$	$\lim_{t\to\infty}\int_{a}^{t}f(x)dx$	
$\int_{-\infty}^{b} f(x) dx$	$lim_{t->-\infty} \int_t^b f(x) dx$	
$\int_{-\infty}^{\infty} f(x) dx$	$\begin{aligned} &\lim_{t\to -\infty} \int_t^c f(x) dx \\ &+ \lim_{t\to \infty} \int_X^t f(x) dx \end{aligned}$	
Conver- gence of $\int f(x) dx$	$\label{eq:limit} \begin{aligned} &\lim {}_{t \to \pm \infty} f(x) dx \text{=} L, \\ &\text{where L is a} \\ &\text{constant} \end{aligned}$	

Parametric Curve C as a function of Parameter t	(x,y)=(f(t),g(t)) for t on [a,b]
Slope at a given point	dy/dx=- $(dy/dt)/(dx/dt)$
Second derivative	$d^2y/dx^2 = (dy/d-t)/(dx/dt)^2$
Polar Curve C as a function of Parameters r and θ	$(r,\theta)=(r,\theta\pm2-$ $\pi n)=(-r,\theta\pm\pi n)$
Slope at a given point	$dy/dx= (dy/d\theta)/(dx/d\theta)$
Cartesian/Rectangular to Polar coordinates	$x=rcos(\theta),$ $y=rsin(\theta)$
Polar to Cartesian/Rectangular coordinates	$r^2=x^2+y^2$ or $r=\sqrt{(x^2+y^2)}$ , $tan\theta=y/x$ or $\theta=arctan(y/x)$

 $(dx/dt)\neq 0$ ,  $(dx/d\theta)\neq 0$ 

# Integral Approximations and Error Bounds

 $\begin{array}{ll} \text{Midpoint} & \Delta x (f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + \ldots + \\ \text{Rule} & _{1}) + f(\bar{x}_n)) \end{array}$ 



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Published 28th May, 2023. Last updated 20th October, 2025. Page 4 of 5.



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# Integral Approximations and Error Bounds (cont)

Trapez-

 $(\Delta x/2)(f(x_1)+2f(x_2)+2f(x_3)+...+2f(x_{n-1})$ 

oidal Rule

 $_{1})+f(x_{1}))$ 

Simpson's  $(\Delta x/3)$ 

Rule

 $(f(x_1)+4f(x_2)+2f(x_3)+4f(x_4)+2f(x_5))+...+$ 

 $2f(x_{n-2})+4f(x_{n-1})+f(x_n)$ 

Midpoint

 $|E_m| \le k(b-a)^3/24n^2$ ,  $k=f''(x)_{max}$  on [a,b]

Rule Error

Bound

Trapez-

 $|E_t| \le k(b-a)^3/12n^2$ ,  $k=f''(x)_{max}$  on [a,b]

oidal Rule

Error

Bound

Simpson's  $|E_S| \le k(b-a)^5/180n^4$ ,  $k=f^{(4)}(x)_{max}$  on

Rule Error

[a,b]

Bound

Integral Approximations are typically used to evaluate an integral that is very difficult or impossible to integrate

 $\Delta x = (b-a)/n$ 

$$\begin{split} \bar{x} = &(x_{i-1} + x_i)/2, \text{ the average/m-} \\ \text{edian of two points } x_{i-1} \text{ and } x_i \end{split}$$

Simpson's Rule can only be used if the given n is even, that is, n=2k for some integer k

In order of most accurate to least accurate approximation: Simpson's Rule, Midpoint Rule, Trapezoidal Rule, Left/Right endpoint approximation



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Published 28th May, 2023. Last updated 20th October, 2025. Page 5 of 5.