

Series				
Series Type	General Summation	Convergence	Divergence	Notes
Infinite Series	$\sum a_n = \sum_{n=k}^{\infty} a_n$, where k is some whole number ($k = \{0, 1, 2, \dots\}$) for which a_n is well-defined	Varies	Varies	
Harmonic Series	$\sum 1/n$	Never converges	Always diverges	The alternating version of this series ($\sum (-1)^{n+1}/n$) converges, and $\sum 1/n$ is a P-Series with $p=1$
Geometric Series	$\sum_{n=0}^{\infty} ar^n = \sum_{n=1}^{\infty} ar^{n-1}$, where a is the first term of the series, and r is the common ratio between terms	Converges if $ r < 1$	Diverges if $ r \geq 1$	If the series converges, its sum is $S = a/(1-r)$
P-Series	$\sum 1/n^p$, where p is a positive number	Converges if $p > 1$	Diverges if $p \leq 1$	
Alternating Series	$\sum (-1)^n b_n$, $\sum (-1)^{n+1} b_n$, or $\sum (-1)^{n-1} b_n$	Converges if $\lim_{n \rightarrow \infty} b_n = 0$ and b_n is a decreasing sequence (or eventually decreasing)	Cannot show divergence, inconclusive	
Telescoping Series	$\sum (b_n - b_{n+1})$	Varies	Varies	
Alternating Series Estimation Theorem: If $S_n = \sum_{i=1}^n (-1)^i b_i$ or $\sum_{i=1}^n (-1)^{i-1} b_i$ is the sum of an alternating series that converges, then $R_n = S - S_n \leq b_{n+1}$ Trigonometric functions like $\cos(n\pi)$ or $\sin(n\pi + \pi/2)$ act as sign alternators, like $(-1)^n$ The Alternating Series Test (AST) checks the limit, but since the AST only concludes convergence, we explicitly apply the Test For Divergence (albeit, redundantly) if the limit fails (even though it checks the same limit)				

Series Tests				
Test Type	Typical series to use test	Convergence	Divergence	Notes
Test for Divergence	$\sum a_n$	Cannot show convergence, inconclusive	Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$ or DNE	
(Direct) Comparison Test	a_n and b_n are positive-termed ($a_n \geq 0$ and $b_n \geq 0$ for all n) and $a_n \leq b_n$ for all n	$\sum a_n$ converges if $\sum b_n$ converges	$\sum b_n$ diverges if $\sum a_n$ diverges	Inconclusive if b_n diverges in proving convergence, or a_n converges in proving divergence



Series Tests (cont)

Limit Comparison Test	a_n and b_n are positive-termed, and $\lim_{n \rightarrow \infty} a_n/b_n = c$, where $0 < c < \infty$	$\sum a_n$ converges $\iff \sum b_n$ converges	$\sum a_n$ diverges $\iff \sum b_n$ diverges	Inconclusive if $c=0$, $c=\infty$, or c DNE
Ratio Test	$\sum a_n$, usually with $n!$ terms, product terms, or $(a_n)^n$	Absolutely converges if $\lim_{n \rightarrow \infty} a_{n+1}/a_n < 1$	Diverges if $\lim_{n \rightarrow \infty} a_{n+1}/a_n > 1$	Inconclusive if $\lim_{n \rightarrow \infty} a_{n+1}/a_n = 1$
Root Test	$\sum a_n$, usually with $(a_n)^n$	Absolutely converges if $\lim_{n \rightarrow \infty} a_n ^{1/n} < 1$	Diverges if $\lim_{n \rightarrow \infty} a_n ^{1/n} > 1$	Inconclusive if $\lim_{n \rightarrow \infty} a_n ^{1/n} = 1$
Integral Test	$a_n = f(x)$, which is a positive, continuous, decreasing function on the interval $[k, \infty)$, usually with clearly-integrable functions	Converges if $\int_k^\infty f(x) dx$ converges	Diverges if $\int_k^\infty f(x) dx$ diverges	
Absolute/Conditional Convergence Classification	$\sum a_n$	Absolutely converges if $\sum a_n $ converges	Diverges if $\sum a_n$ diverges	Conditionally converges if $\sum a_n $ diverges, but $\sum a_n$ converges

For the series listed, assume each series to be an infinite series starting at $n=k$: $\sum_{n=k}^\infty = \sum$ as previously defined

If a test is inconclusive, use another test

The symbol $[\iff]$ represents the relationship "if and only if" (often abbreviated to "iff"), meaning both sides of the statement must be true at the same time, or false at the same time

Special Series

Series	Summation Form	First five terms	Radius & Interval of Convergence
Power Series centered at a	$\sum C_n(x-a)^n$	$C_0 + C_1(x-a) + C_2(x-a)^2 + C_3(x-a)^3 + C_4(x-a)^4 + \dots$	$(a-R, a+R)$, $[a-R, a+R)$, $(a-R, a+R]$, or $[a-R, a+R]$
Taylor Series centered at a	$\sum f^{(n)}(a)(x-a)^n/n!$	$f(a) + f'(a)(x-a) + f''(a)(x-a)^2/2! + f'''(a)(x-a)^3/3! + f^{(4)}(a)(x-a)^4/4! + \dots$	$ x-a < R$
Maclaurin Series (Taylor Series centered at 0)	$\sum f^{(n)}(0)(x-0)^n/n! = \sum f^{(n)}(0)x^n/n!$	$f(0) + f'(0)x + f''(0)x^2/2! + f'''(0)x^3/3! + f^{(4)}(0)x^4/4! + \dots$	$ x < R$
$1/(1-x)$	$\sum x^n$	$1 + x + x^2 + x^3 + x^4 + \dots$	$R=1$, $I=(-1, 1)$
e^x	$\sum x^n/n!$	$1 + x + x^2/2! + x^3/3! + x^4/4! + \dots$	$R=\infty$, $I=(-\infty, \infty)$
$\ln(1+x)$	$\sum (-1)^n x^{n+1}/(n+1)$	$x - x^2/2 + x^3/3 - x^4/4 + x^5/5 - \dots$	$R=1$, $I=(-1, 1]$
$\arctan(x)$	$\sum (-1)^n x^{2n+1}/(2n+1)$	$x - x^3/3 + x^5/5 - x^7/7 + x^9/9 - \dots$	$R=1$, $I=[-1, 1]$



Special Series (cont)

$\sin(x)$	$\sum (-1)^n x^{2n+1} / (2n+1)!$	$x - x^3/3! + x^5/5! - x^7/7! + x^9/9! - \dots$	$R = \infty, I = (-\infty, \infty)$
$\cos(x)$	$\sum (-1)^n x^{2n} / (2n)!$	$1 - x^2/2! + x^4/4! - x^6/6! + x^8/8! - \dots$	$R = \infty, I = (-\infty, \infty)$
$(1+x)^k$	$\sum \binom{k}{n} x^n = \sum ((k(k-1)(k-2)\dots(k-n+1))/n!) x^n$	$1 + kx + k(k-1)x^2/2! + k(k-1)(k-2)x^3/3! + k(k-1)(k-2)(k-3)x^4/4! + \dots$	$R = 1$

Taylor's Inequality $|R_n(x)| \leq M|x-a|^{n+1}/(n+1)!$, where M is a constant such that $|f^{(n+1)}(x)| \leq M$ for all $|x-a| \leq d$

For the series listed, assume each series to be an infinite series starting at $n=0$: $\sum_{n=0}^{\infty} = \Sigma$ as previously defined

The Radius of Convergence, R, is typically found by using the Ratio Test or Root Test

$\binom{k}{n}$ is the "binomial coefficient" (read as "k choose n"). $\binom{k}{n} = k!/(n!(k-n)!)$

$f^{(n)}$ means "the nth derivative of the function f"

$n! = n(n-1)! = n(n-1)(n-2)! = n(n-1)(n-2)(n-3)! = \dots$

$n! = n(n-1)(n-2)(n-3)\dots \cdot 3 \cdot 2 \cdot 1$

$0! = 1, 1! = 1$

Areas of Functions	Surface Areas	Trigonometric Integrals	Improper Integrals (cont)
Between two functions $\int_a^b ((\text{top function}) - (\text{bottom function})) dx$	Function revolved about an axis $2\pi \int_a^b (\text{radius})(\text{Arc Length component}) ds$	https://cheatography.com/crossant/cheat-sheets/integral-trigonometry/	Divergence of $\int f(x) dx$ $\lim_{t \rightarrow \pm\infty} f(x) dx = \pm\infty$ or DNE
Enclosed by a polar function $\frac{1}{2} \int_a^b f(\theta)^2 d\theta$	Function revolved about y-axis $2\pi \int_a^b x \sqrt{1 + (f'(x))^2} dx$	Integration by Partial Fractions $(ax+b)/(px+q)(rx+s)$ $A/(px+q) + B/(rx+s)$	Improper Integrals are integrals with bounds at infinity (Type 1) or at least one discontinuity on the integrated region (Type 2)
Between two polar functions $\frac{1}{2} \int_a^b ((\text{outer polar function})^2 - (\text{inner polar function})^2) d\theta$	Function revolved about x-axis $2\pi \int_a^b y \sqrt{1 + (g'(y))^2} dy$	$(ax+b)/(px+q)^2$ $A/(px+q) + B/(px+q)^2$	Conic Sections https://cheatography.com/crossant/cheat-sheets/conic-sections/
Area enclosed by a polar function is with respect to the pole, which is the origin	Parametric function of t revolved about y-axis $2\pi \int_a^b f(x) \sqrt{(x'(t))^2 + (y'(t))^2} dt$	$(ax^2+bx+c)/((px+q)(rx^2+sx+t))$ $A/(px+q) + B/(rx^2+sx+t) + Cx+D/(rx^2+sx+t)^2$	Parametric Curves and Polar Functions
Average value of a function on an interval: $f_{avg} = 1/(b-a) \int_a^b f(x) dx$	Parametric function of t revolved about x-axis $2\pi \int_a^b g(y) \sqrt{(x'(t))^2 + (y'(t))^2} dt$	For numerator/denominator N/D, partial fractions simplify integrals of polynomial rational expressions where $\deg(N) < \deg(D)$, and a fully-factored D, into a sum of simpler fractions. If $\deg(N) \geq \deg(D)$, then polynomial long division must be used beforehand.	
Volumes of Solids of Revolution	Integration by Parts		
Disk $\pi \int_a^b (\text{radius})^2 dr$	Indefinite Integral $\int u dv = uv - \int v du$		
Washer $\pi \int_a^b ((\text{outer radius})^2 - (\text{inner radius})^2) dr$	Definite Integral $\int_a^b u dv = uv _a^b - \int_a^b v du$		
Cylindrical Shell $2\pi \int_a^b (\text{radius})(\text{height}) dr$	Integration by Parts is used to integrate integrals that have components multiplied together in their simplest form, often referred to as a "product rule for integrals"		
Disk: slices are perpendicular and connected to axis of rotation Washer: slices are perpendicular and disconnected to axis of rotation Cylindrical Shells: slices are parallel to axis of rotation	Choosing the "dv" term depends		
Arc Lengths			

Function	$\int_a^b \sqrt{1+(f'(x))^2} dx$	on what will simplify the integral the best, while being relatively simple to integrate	Improper Integrals	Parametric Curve C as a function of Parameter t
Parametric Function	$\int_a^b \sqrt{(x'(t))^2+(y'(t))^2} dt$	The constant of integration does not need to be inserted until the integral has been fully simplified	$\int_a^\infty f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$	Slope at a given point
Polar Function	$\int_a^b \sqrt{(r(\theta))^2+(r'(\theta))^2} d\theta$		$\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$	Second derivative
For standard functions: $f'(x)=-dy/dx$ For parametric functions: $x'(t)=-dx/dt$ and $y'(t)=dy/dt$ For polar functions: $r'(\theta)=dr/d\theta$			$\int_{-\infty}^\infty f(x) dx = \lim_{t \rightarrow -\infty} \int_t^c f(x) dx + \lim_{t \rightarrow \infty} \int_c^t f(x) dx$	Polar Curve C as a function of Parameters r and θ
			Convergence of $\int f(x) dx$	Slope at a given point
			$\lim_{t \rightarrow \pm\infty} \int f(x) dx = L$, where L is a constant	Cartesian/Rectangular to Polar coordinates
				Polar to Cartesian/Rectangular coordinates
				$(dx/dt) \neq 0, (dx/d\theta) \neq 0$
				Integral Approximations and Error Bounds
				Midpoint Rule



Integral Approximations and Error Bounds (cont)

Trapezoidal Rule $(\Delta x/2)(f(x_1)+2f(x_2)+2f(x_3)+\dots+2f(x_{n-1})+f(x_n))$

Simpson's Rule $(\Delta x/3)(f(x_1)+4f(x_2)+2f(x_3)+4f(x_4)+2f(x_5)+\dots+2f(x_{n-2})+4f(x_{n-1})+f(x_n))$

Midpoint Rule Error Bound $|E_M| \leq k(b-a)^3/24n^2$, $k=f''(x)_{\max}$ on $[a,b]$

Trapezoidal Rule Error Bound $|E_T| \leq k(b-a)^3/12n^2$, $k=f''(x)_{\max}$ on $[a,b]$

Simpson's Rule Error Bound $|E_S| \leq k(b-a)^5/180n^4$, $k=f^{(4)}(x)_{\max}$ on $[a,b]$

Integral Approximations are typically used to evaluate an integral that is very difficult or impossible to integrate

$$\Delta x = (b-a)/n$$

$\bar{x} = (x_{i-1} + x_i)/2$, the average/midpoint of two points x_{i-1} and x_i

Simpson's Rule can only be used if the given n is even, that is, $n=2k$ for some integer k

In order of most accurate to least accurate approximation:
Simpson's Rule, Midpoint Rule, Trapezoidal Rule, Left/Right endpoint approximation

