

Notation						
Name	Operation	y versions	f(x) versions	Composition versions	Second derivative	nth derivative
Leibniz/Fraction Notation	$d/dx (f(x))=d/dx (y)$	$dy/dx=dy(x)/dx$	$df/dx=df(x)/dx=d(f(x))/dx$	$df/dg*dg/dx$	$d^2f/dx^2$	$d^n/dx^n=d^n f/dx^n$
Lagrange/Prime Notation	$d/dx (f(x))=d/dx (y)$	$y'$	$f'=f'(x)=(f(x))'$	$(f(g(x)))'$	$f''$	$f^{(n)}(x)$
Newton/Dot Notation	$d/dt (f(t))=d/dt (y(t))$	$\dot{y}$			$\ddot{y}$	
Euler/D-Notation	$D_x(f)$	$D_x y$	$D_x f$	$D_x(f(g))$	$D_x^2 f$	$D_x^n f$

$n \in \mathbb{N}_1 = \{1,2,3,4,5,\dots\}$

Derivative Rules			
Formal/Limit Definition of a Derivative	$f'(x)=\lim_{h \rightarrow 0} (f(x+h)-f(x))/h$		
Limit Definition of the Derivative at a point	$f'(a)=\lim_{h \rightarrow 0} (f(a+h)-f(a))/h$	$f'(a)=\lim_{x \rightarrow a} (f(x)-f(a))/(x-a)$	
Linearity 1: Constant-Multiple Rule	$d/dx (kf(x))$	$k*d/dx (f)$	$kf'$
Linearity 2: Sum/Difference Rule	$d/dx (f(x)\pm g(x))$	$d/dx (f) \pm d/dx (g)$	$f' \pm g'$
Product Rule	$d/dx (f(x)*g(x))$	$f'g+fg'$	
Multi-Product Rule	$d/dx (p(x)*q(x)*r(x)*s(x)*\dots)$	$p'qrs\dots + pq'r's\dots + pqr's'\dots + pqr's'\dots*(p'/p + q'/q + r'/r + s'/s + \dots)$	
Quotient Rule	$d/dx (f(x)/g(x))$	$(f'g-fg')/g^2$	$g(x)\neq 0$ , quotients can be rewritten into products with sign-flipped exponents
Chain Rule	$d/dx (f(g(x)))$	$f'(g)g'$	
Multi-Chain Rule	$d/dx (p(q(r(s(\dots))))))$	$p'(q(r(s(\dots))))*q'(r(s(\dots)))*r'(s(\dots))*s'(\dots)*\dots$	
First Fundamental Theorem of Calculus (FTC I)	$d/dx (\int_a^x f(t)dt)$	$f(x)$	Derivatives and integrals are inverses of each other
FTC I Chain Rule 1	$d/dx (\int_a^{v(x)} f(t)dt)$	$f(v)v'$	
FTC I Chain Rule 2	$d/dx (\int_{u(x)}^{v(x)} f(t)dt)$	$f(v)v'-f(u)u'$	
Summation Rule	$d/dx (\Sigma f(x))$	$\Sigma f'(x)$	The summation must be within its interval of convergence

a and k are constants

f, g, p, q, r, s, u, and v are functions of x such that  $f=f(x)$ ,  $g=g(x)$ ,  $p=p(x)$ ,  $q=q(x)$ ,  $r=r(x)$ ,  $s=s(x)$ ,  $u=u(x)$ , and  $v=v(x)$ , unless otherwise shown

Derivatives of Algebraic Functions						
Rule	Function Derivative	Derivative	nth Derivative of Function	nth Derivative	Function Composition	Derivative by Chain Rule



### Derivatives of Algebraic Functions (cont)

Constant	$d/dx (k)$	0	$d^n/dx^n (k)$	0	$d/dx (f(k))$	0
Power	$d/dx (x^k)$	$kx^{k-1}$	$d^n/dx^n (x^k), k \neq 0, k-n+1 \neq 0$	$\Gamma(k+1)x^{k-n}/\Gamma(k-n+1)$	$d/dx (u(x)^k), u(x) \neq 0$	$ku^{k-1}u'$
Natural Exponential	$d/dx (e^x)$	$e^x$	$d^n/dx^n (e^x)$	$e^x$	$d/dx (e^{u(x)})$	$e^u u'$
Natural Logarithm	$d/dx (\ln(x))$	$1/x$	$d^n/dx^n (\ln(x))$	$(-1)^{n+1}(n-1)!/x^n$	$d/dx (\ln(u(x))), u(x) > 0$	$u'/u$
General Exponential	$d/dx (k^x), k > 0$	$k^x \ln(k)$	$d^n/dx^n (k^x), k > 0$	$k^x (\ln(k))^n$	$d/dx (k^{u(x)}), k > 0$	$k^u \ln(k) u'$
General Logarithm	$d/dx (\log_k(x)), k > 0, k \neq 1$	$1/(x \ln(k))$	$d^n/dx^n (\log_k(x)), k > 0, k \neq 1$	$(-1)^{n+1}(n-1)!/(x^n \ln(k))$	$d/dx (\log_k(u(x))), k > 0, k \neq 1, u(x) \neq 0$	$u'/(u \ln(k))$
Absolute Value	$d/dx ( x )$	$x/ x $			$d/dx ( u(x) ), u(x) \neq 0$	$u' * u/ u $
Function-Power-Function	$d/dx (f(x)^{g(x)}, f(x) > 0)$	$f^g (f^g/f + \ln(f)g')$				

k is a constant

f=f(x), g=g(x), and u=u(x) are all functions of the variable x

m, n ∈ ℕ<sub>1</sub> = {1,2,3,4,5,...}

Γ(x) is the gamma function, which defines factorials for negative/non-integer numbers

x! = Γ(x+1)

n! = n(n-1)!(n-1)(n-2)!(n-1)(n-2)(n-3)! = ...

n! = n(n-1)(n-2)(n-3)...\*3\*2\*1

0!=1, 1!=1

### Derivatives of Trigonometric Functions

Standard Trigonometric	Derivative	Inverse Trigonometric	Derivative	Hyperbolic Trigonometric	Derivative	Hyperbolic Inverse Trigonometric	Derivative
$d/dx (\sin(x))$	$\cos(x)$	$d/dx (\arcsin(x))$	$1/\sqrt{1-x^2}$	$d/dx (\sinh(x))$	$\cosh(x)$	$d/dx (\operatorname{arcsinh}(x))$	$1/\sqrt{1+x^2}$
$d/dx (\cos(x))$	$-\sin(x)$	$d/dx (\arccos(x))$	$-1/\sqrt{1-x^2}$	$d/dx (\cosh(x))$	$\sinh(x)$	$d/dx (\operatorname{arccosh}(x))$	$-1/\sqrt{x^2-1}$
$d/dx (\tan(x))$	$\sec^2(x)$	$d/dx (\arctan(x))$	$1/(1+x^2)$	$d/dx (\tanh(x))$	$\operatorname{sech}^2(x)$	$d/dx (\operatorname{arctanh}(x))$	$1/(1-x^2)$
$d/dx (\csc(x))$	$-\csc(x)\cot(x)$	$d/dx (\operatorname{arccsc}(x))$	$-1/( x \sqrt{x^2-1})$	$d/dx (\operatorname{csch}(x))$	$-\operatorname{csch}(x)\operatorname{coth}(x)$	$d/dx (\operatorname{arccsch}(x))$	$-1/( x \sqrt{x^2+1})$
$d/dx (\sec(x))$	$\sec(x)\tan(x)$	$d/dx (\operatorname{arcsec}(x))$	$1/( x \sqrt{x^2-1})$	$d/dx (\operatorname{sech}(x))$	$-\operatorname{sech}(x)\operatorname{tanh}(x)$	$d/dx (\operatorname{arcsech}(x))$	$-1/( x \sqrt{1-x^2})$
$d/dx (\cot(x))$	$-\csc^2(x)$	$d/dx (\operatorname{arccot}(x))$	$-1/(1+x^2)$	$d/dx (\operatorname{coth}(x))$	$-\operatorname{csch}^2(x)$	$d/dx (\operatorname{arccoth}(x))$	$1/(1-x^2)$

$d^n/dx^n (\sin(x)) = \sin(x+n\pi/2)$

$d^n/dx^n (\cos(x)) = \cos(x+n\pi/2)$

$\sinh(x) = (e^x - e^{-x})/2$

$\cosh(x) = (e^x + e^{-x})/2$

$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2+1})$

$\operatorname{arccosh}(x) = \ln(x + \sqrt{x^2-1}), x \geq 1$

### Polynomial Derivative Examples

### Trigonometric Derivative Examples (cont)

$$d/dx (\sin(\arccos(x))) = -x/\sqrt{1-x^2}$$

$$d/dx (\sin(k)) = 0$$

### Exponential Derivative Examples

$d/dx (x)$	1
$d/dx (x^2)$	2x
$d/dx (x^3)$	3x <sup>2</sup>
$d/dx (x^4)$	4x <sup>3</sup>
$d/dx (1/x)$	-1/x <sup>2</sup>
$d/dx (-1/x^2)$	2/x <sup>3</sup>
$d/dx (2/x^3)$	-6/x <sup>4</sup>
$d/dx (-6/x^4)$	24/x <sup>5</sup>
$d/dx (\sqrt{x})$	1/(2√x)
$d/dx (x^{1/3})$	1/(3x <sup>2/3</sup> )
$d/dx (x^{1/4})$	1/(4x <sup>3/4</sup> )
$d/dx (x^{3/2})$	3√x/2
$d/dx (x^{5/3})$	5x <sup>2/3</sup> /3
$d/dx (x^{-2-3})$	(-√2-3)x <sup>-√2-4</sup>
$d/dx (1/(1+x))$	-1/(1+x) <sup>2</sup>
$d/dx (-1/(1+x)^2)$	2/(1+x) <sup>3</sup>
$d/dx (-1/(1-x))$	-1/(1-x) <sup>2</sup>
$d/dx (-1/(1-x)^2)$	-2/(1-x) <sup>3</sup>
$d/dx (\sqrt{5x+1})$	5/(2√(4x+1))
$d/dx (\sqrt{x^5+1})$	5x <sup>4</sup> /(2√(x <sup>5</sup> +1))
$d/dx ((2x^2+5)^9)$	36x(2x <sup>2</sup> +5) <sup>8</sup>
<b>d/dx (1)</b>	<b>0</b>

$d/dx (xe^x)$	e <sup>x</sup> +xe <sup>x</sup>
$d/dx (e^{2x})$	2e <sup>2x</sup>
$d/dx (e^{x^2})$	2xe <sup>x<sup>2</sup></sup>
$d/dx (e^{e^x})$	e <sup>x</sup> e <sup>e<sup>x</sup></sup>
$d/dx (x^x)$	x <sup>x</sup> (ln(x)+1)
$d/dx (2^{3^x})$	2 <sup>3<sup>x</sup></sup> 3 <sup>x</sup> *ln(2)*ln(3)
<b>d/dx (e<sup>k</sup>)</b>	<b>0</b>

#### Logarithmic Derivative Examples

$d/dx (\ln(1/x))$	-1/x
$d/dx (\ln(1+x))$	1/(1+x)
$d/dx (\ln(1-x))$	-1/(1-x)
<b>d/dx (ln(x<sup>2</sup>))</b>	<b>2/x</b>
$d/dx (\ln(x^3))$	3/x
$d/dx (\ln(x^4))$	4/x
$d/dx (x \ln(x))$	ln(x)+1
$d/dx (\ln(\ln(x)))$	1/(x ln(x))
<b>d/dx (ln(k))</b>	<b>0</b>

#### Special/Other Derivative Examples

$d/dx (e^x \sin(x))$	e <sup>x</sup> sin(x)+e <sup>x</sup> cos(x)
$d/dx (e^x \cos(x))$	e <sup>x</sup> cos(x)-e <sup>x</sup> sin(x)
$d/dx (\sin^x(x))$	sin <sup>x</sup> (x)(ln(sin(x))+xcot(x))
$d/dx (\sin(x)^{\cos(x)})$	sin(x) <sup>cos(x)</sup> (cos <sup>2</sup> (x)csc(x)-sin(x)ln(sin(x)))
$d/dx (\ln(1/(1-x)))$	1/(1-x)
<b>d/dx (ln(x<sup>3</sup>+7x+12))</b>	<b>(3x<sup>2</sup>+7)/(x<sup>3</sup>+7x+12)</b>
$d/dx (\ln(e^{3x} \tan(x^3)))$	3+(3x <sup>2</sup> sec <sup>2</sup> (x <sup>3</sup> ))/(tan(x <sup>3</sup> ))
<b>d/dx (1+k+t+√2+cos(a)+e+π+-ln(3))</b>	<b>0</b>

#### Trigonometric Derivative Examples

$d/dx (-\sin(x))$	-cos(x)
$d/dx (-\cos(x))$	sin(x)
$d/dx (\sin(2x))$	2cos(2x)
$d/dx (\cos(2x))$	-2sin(2x)
$d/dx (\sin^2(x))$	2sin(x)cos(x)
$d/dx (\cos^2(x))$	-2cos(x)sin(x)
<b>d/dx (arctan(3x))</b>	<b>3/(1+9x<sup>2</sup>)</b>
$d/dx (\sin(\sin(x)))$	cos(x)cos(sin(x))

