Calculus Derivatives and Differentiation Cheat Sheet

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Notation

Name	Operation	y versions	f(x) versions	Composition versions	Second derivative	nth derivative
Leibniz/Fraction Notation	d/dx (f(x))=d/dx (y)	**dy/dx=dy(x)/dx**	df/dx=df(x)/dx=d(f-(x))/dx	**df/dg*dg/dx**	**d^2f/dx^2**	**d^n/dx^n**$=d^nf/dx^n$
Lagrange/Prime Notation	d/dx (f(x))=d/dx (y)	**y'**	**f'=f'(x)**=(f(x))'	(f(g(x)))'	**f''**	**$f^{(n)}(x)$**
Newton/Dot Notation	d/dt (f(t))=d/dt (y(t))	ẏ			ÿ	
Euler/D-Notation	$D_x(f)$	D_xy	D_xf	$D_x(f(g))$	D_x^2f	D_x^nf

$n \in \mathbb{N}_1 = \{1,2,3,4,5,...\}$

Derivative Rules

Formal/Limit Definition of a Derivative	f'(x)=lim $_{h->0}$ (f(x+h)-f(x))/h		
Limit Definition of the Derivative at a point	f'(a)=lim $_{h->0}$ (f(a+h)-f(a))/h	f'(a)=lim $_{x->a}$ (f(x)-f(a))/(x-a)	
Linearity 1: Constant-Multiple Rule	**d/dx (kf(x))**	**k*d/dx (f)**	**kf'**
Linearity 2: Sum/Difference Rule	**d/dx (f(x)±g(x))**	**d/dx (f) ± d/dx (g)**	**f'±g'**
Product Rule	**d/dx (f(x)*g(x))**	**f'g+fg'**	
Multi-Product Rule	d/dx (p(x)*q(x)*r-(x)*s(x)*...)	p'qrs... + pq'rs... + pqr's... + pqrs'... + ...	pqrs...*(p'/p + q'/q + r'/r + s'/s + ...)
Quotient Rule	d/dx (f(x)/g(x))	$(f'g-fg')/g^2$	g(x)≠0, quotients can be rewritten into products with sign-flipped exponents
Chain Rule	**d/dx (f(g(x)))**	**f'(g)g'**	
Multi-Chain Rule	**d/dx (p(q(r-(s(...)))))**	**p'(q(r(s(...))))*q'(r(s(...)))*r'(s(...))*s'(...)*...**	
First Fundamental Theorem of Calculus (FTC I)	**d/dx ($\int_a^x$ f(t)dt)**	**f(x)**	**Derivatives and integrals are inverses of each other**
FTC I Chain Rule 1	d/dx ($\int_a^{v(x)}$ f(t)dt)	f(v)v'	
FTC I Chain Rule 2	d/dx ($\int_{u(x)}^{v(x)}$ f(t)dt)	f(v)v'-f(u)u'	
Summation Rule	d/dx (Σf(x))	Σf'(x)	The summation must be within its interval of convergence

a and k are constants

f, g, p, q, r, s, u, and v are functions of x such that f=f(x), g=g(x), p=p(x), q=q(x), r=r(x), s=s(x), u=u(x), and v=v(x), unless otherwise shown

Derivatives of Algebraic Functions

Rule	Function Derivative	Derivative	nth Derivative of Function	nth Derivative	Function Composition	Derivative by Chain Rule

Derivatives of Algebraic Functions (cont)

Constant	**d/dx (k)**	**0**	d^n/dx^n (k)	0	d/dx (f(k))	0
Power	**d/dx (x^k)**	**kx^{k-1}**	d^n/dx^n (x^k), *k≠0, k-n+1≠-n*	$\Gamma(k+1)x^{k-n}/\Gamma(k-n+1)$	**d/dx ($u(x)^k$)**, *u(x)≠0*	**$ku^{k-1}u'$**
Natural Exponential	**d/dx (e^x)**	**e^x**	d^n/dx^n (e^x)	e^x	**d/dx ($e^{u(x)}$)**	**$e^u u'$**
Natural Logarithm	**d/dx (ln(x))**	**1/x**	d^n/dx^n (ln(x))	$(-1)^{n+1}(n-1)!/x^n$	**d/dx (ln(u(x)))**, *u(x)>0*	**u'/u**
General Exponential	d/dx (k^x), *k>0*	k^xln(k)	d^n/dx^n (k^x), *k>0*	$k^x(\ln(k))^n$	d/dx ($k^{u(x)}$), *k>0*	k^uln(k)u'
General Logarithm	d/dx ($\log_k$(x)), *k>0, k≠1*	1/(xln(k))	d^n/dx^n ($\log_k$(x)), *k>0, k≠1*	$(-1)^{n+1}(n-1)!/(x^n\ln(k))$	d/dx ($\log_k$(u(x))), *k>0, k≠1, u(x)≠0*	u'/(uln(k))
Absolute Value	d/dx (|x|)	x/|x|			d/dx (|u(x)|), *u(x)≠0*	u'*u/|u|
Function-Power-Function	d/dx ($f(x)^{g(x)}$), *f(x)>0*	f^g(f'g/f+ln(f)g')				

k is a constant

f=f(x), g=g(x), and u=u(x) are all functions of the variable x

m, n ∈ $\mathbb{N}_1$ = {1,2,3,4,5,...}

Γ(x) is the gamma function, which defines factorials for negative/non-integer numbers

x! = Γ(x+1)

n! = n(n-1)!=n(n-1)(n-2)!=n(n-1)(n-2)(n-3)!=...

n! = n(n-1)(n-2)(n-3)...*3*2*1

0!=1, 1!=1

Derivatives of Trigonometric Functions

Standard Trigonometric	Derivative	Inverse Trigonometric	Derivative	Hyperbolic Trigonometric	Derivative	Hyperbolic Inverse Trigonometric	Derivative
d/dx (sin(x))	**cos(x)**	**d/dx (arcsin(x))**	**$1/\sqrt{(1-x^2)}$**	d/dx (sinh(x))	cosh(x)	d/dx (arcsinh(x))	$1/\sqrt{(1+x^2)}$
d/dx (cos(x))	**-sin(x)**	d/dx (arccos(x))	$-1/\sqrt{(1-x^2)}$	d/dx (cosh(x))	sinh(x)	d/dx (arccosh(x))	$-1/\sqrt{(x^2-1)}$
d/dx (tan(x))	**$\sec^2$(x)**	**d/dx (arctan(x))**	**$1/(1+x^2)$**	d/dx (tanh(x))	sech^2(x)	d/dx (arctanh(x))	$1/(1-x^2)$
d/dx (csc(x))	-csc(x)cot(x)	d/dx (arccsc(x))	$-1/(\|x\|\sqrt{(x^2-1)})$	d/dx (csch(x))	-csch(x)coth(x)	d/dx (arccsch(x))	$-1/(\|x\|\sqrt{(x^2+1)})$
d/dx (sec(x))	**sec(x)tan(x)**	d/dx (arcsec(x))	$1/(\|x\|\sqrt{(x^2-1)})$	d/dx (sech(x))	-sech(x)tanh(x)	d/dx (arcsech(x))	$-1/(\|x\|\sqrt{(1-x^2)})$
d/dx (cot(x))	$-\csc^2$(x)	d/dx (arccot(x))	$-1/(1+x^2)$	d/dx (coth(x))	$-\text{csch}^2$(x)	d/dx (arccoth(x))	$1/(1-x^2)$

d^n/dx^n (sin(x)) = sin(x+nπ/2)

d^n/dx^n (cos(x)) = cos(x+nπ/2)

sinh(x) = (e^x-e^{-x})/2

cosh(x) = (e^x+e^{-x})/2

arcsinh(x) = $\ln(x+\sqrt{(x^2+1)})$

arccosh(x) = $\ln(x+\sqrt{(x^2-1)})$, x≥1

Polynomial Derivative Examples

Trigonometric Derivative Examples (cont)

d/dx (sin(arccos(x)))	$-x/\sqrt{(1-x^2)}$
d/dx (sin(k))	**0**

Exponential Derivative Examples

d/dx (x)	**1**
d/dx (x^2)	**2x**
d/dx (x^3)	**$3x^2$**
d/dx (x^4)	$4x^3$
d/dx (1/x)	**$-1/x^2$**
d/dx ($-1/x^2$)	**$2/x^3$**
d/dx ($2/x^3$)	$-6/x^4$
d/dx ($-6/x^4$)	$24/x^5$
d/dx (√x)	**1/(2√x)**
d/dx ($x^{1/3}$)	**$1/(3x^{2/3})$**
d/dx ($x^{1/4}$)	$1/(4x^{3/4})$
d/dx ($x^{3/2}$)	3√x/2
d/dx ($x^{5/3}$)	$5x^{2/3}/3$
d/dx ($x^{-\sqrt{2}-3}$)	**$(-\sqrt{2}-3)x^{-\sqrt{2}-4}$**
d/dx (1/(1+x))	$-1/(1+x)^2$
d/dx ($-1/(1+x)^2$)	$2/(1+x)^3$
d/dx (-1/(1-x))	$-1/(1-x)^2$
d/dx ($-1/(1-x)^2$)	$-2/(1-x)^3$
d/dx (√(5x+1))	5/(2√(4x+1))
d/dx ($\sqrt{(x^5+1)}$)	$5x^4/(2\sqrt{(x^5+1)})$
d/dx ($(2x^2+5)^9$)	$36x(2x^2+5)^8$
d/dx (1)	**0**

Special/Other Derivative Examples

d/dx ($e^x\sin(x)$)	$e^x\sin(x)+e^x\cos(x)$
d/dx ($e^x\cos(x)$)	$e^x\cos(x)-e^x\sin(x)$
d/dx ($\sin^x(x)$)	$\sin^x(x)(\ln(\sin(x))+x\cot(x))$
d/dx ($\sin(x)^{\cos(x)}$)	$\sin(x)^{\cos(x)}(\cos^2(x)\csc(x)-\sin(x)\ln(\sin(x)))$
d/dx (ln(1/(1-x)))	1/(1-x)
d/dx ($\ln(x^3+7x+12)$)	**$(3x^2+7)/(x^3+7x+12)$**
d/dx ($\ln(e^{3x}\tan(x^3))$)	$3+(3x^2\sec^2(x^3))/(\tan(x^3))$
d/dx (1+k+t+√2+cos(a)+e+π+ln(3))	**0**

Trigonometric Derivative Examples

d/dx (-sin(x))	**-cos(x)**
d/dx (-cos(x))	**sin(x)**
d/dx (sin(2x))	**2cos(2x)**
d/dx (cos(2x))	**-2sin(2x)**
d/dx ($\sin^2(x)$)	2sin(x)cos(x)
d/dx ($\cos^2(x)$)	-2cos(x)sin(x)
d/dx (arctan(3x))	**$3/(1+9x^2)$**
d/dx (sin(sin(x)))	cos(x)cos(sin(x))

d/dx (xe^x)	e^x+xe^x
d/dx (e^{2x})	**$2e^{2x}$**
d/dx (e^{x^2})	**$2xe^{x^2}$**
d/dx (e^{e^x})	$e^xe^{e^x}$
d/dx (x^x)	$x^x(\ln(x)+1)$
d/dx (2^{3^x})	$2^{3^x}*3^x*\ln(2)*\ln(3)$
d/dx (e^k)	**0**

Logarithmic Derivative Examples

d/dx (ln(1/x))	-1/x
d/dx (ln(1+x))	1/(1+x)
d/dx (ln(1-x))	-1/(1-x)
d/dx ($\ln(x^2)$)	**2/x**
d/dx ($\ln(x^3)$)	3/x
d/dx ($\ln(x^4)$)	4/x
d/dx (xln(x))	ln(x)+1
d/dx (ln(ln(x)))	1/(xln(x))
d/dx (ln(k))	**0**