

CSCI 2020 Computer Architecture

Basic binary math and logic

Binary numbers operate in base 2, where the only digits are 0 and 1. They are fundamental in digital systems, logic design, and computing.

1. Binary Arithmetic: Addition:

Binary addition is similar to decimal addition, with the rules:

2. Binary Subtraction: two's complement(invert bits add 1) of

what is subtracting, add what is being taken from and discard carry

Logical Operations:

- **AND** ($A * B$): Outputs 1 only if both inputs are 1
- **OR** ($A + B$): Outputs 1 if at least one input is 1.
- **XOR** ($A \oplus B$): Outputs 1 if the inputs are different.
- **NOT** ($\neg A$): Inverts the input.
- **NAND** ($\neg(A * B)$): Outputs 1 unless both inputs are 1.
- **NOR** ($\neg(A + B)$): Outputs 1 only if both inputs are 0.
- **XNOR** ($\neg(A \oplus B)$): Outputs 1 if the inputs are the same.
- **Implication** ($A \rightarrow B$): Outputs 1 unless $A = 1$ and $B = 0$.

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- **Equivalence** ($A \leftrightarrow B$): Outputs 1 if A and B are equal.

Equivalent to XNOR.

Total bit Combinations = 2^n

Dec \rightarrow **Hex** | Divide by 16, use remainders as hex digits. | 156 (dec) | 9C (hex)

Hex \rightarrow **Dec** | Multiply digits by 16^n , sum the results. | 1A3 (hex) | 419 (dec)

Bin \rightarrow **Dec** | Multiply bits by 2^n , sum the results. | 1101 (bin) | 13 (dec)

Dec \rightarrow **Oct** | Divide by 8, use remainders as octal digits. | 125 (dec) | 175 (oct)

Bin \rightarrow **Oct** | Group binary into 3 bits, convert each group to octal. | 110101 (bin) | 65 (oct)

Sign Representation MSB(the front of bit): 0 (positive), 1 (negative) Negative: Invert and add 1 | | **Arithmetic** | Complex | Simplified (no separate subtraction) | | **Range (4-bit)** | (signed Magnitude)(-7 to $+7$) | (two's complement)(-8 to $+7$) |

Combinational Logic

Combinational logic

Definition

- **Combinational Circuit**: Output depends only on current inputs; no memory.

- **Sequential Circuit**: Output depends on current inputs and past states (memory)

Key Differences

| Feature | Combinational Circuit | Sequential Circuit |

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| **Output Dependency** | Current inputs only | Current inputs + past states |

| **Memory** | No memory | Has memory (e.g., flip-flops) |

| **Feedback** | No feedback loop | Includes feedback loop |

| **Clock Signal** | Not required | Requires a clock signal |

| **Time Dependency** | Outputs appear immediately | Outputs depend on clock cycles |

Examples

- **Combinational Circuit**:

Combinational Logic (cont)

- **Half Adder**: Adds two inputs, outputs Sum = $(A \oplus B)$, Carry = $(A \text{ AND } B)$.

- **Sequential Circuit**:

- **D Flip-Flop**: Stores 1 bit of data; updates on clock edge.

Applications

- **Combinational Circuits**:

- Adders, multiplexers, encoders, decoders.

- **Sequential Circuits**:

- Counters, shift registers, memory units.

Summary

- **Combinational Circuits**:

- Simple, stateless, used for logical operations.

- **Sequential Circuits**:

- Complex, state-based, used for time-sensitive or memory-based tasks.

For AND(also minterm) gates

- **Identity**: AND-ing anything with 1 keeps it the same.

- **Null**: AND-ing anything with 0 makes it 0.

- **Commutative**: Changing the order of inputs doesn't matter.

- **Associative**: Grouping inputs in any way doesn't matter.

- **Minterm** = $A \cdot B \cdot C \cdot D$



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Combinational Logic (cont)

- **sum of products:** $Y = (A \cdot B \cdot C \cdot D) + (A \cdot B \cdot C \cdot D) + (A \cdot B \cdot C \cdot D)$

For OR(maxterm) gate

- **Identity:** $(A + 0 = A)$

- **Null:** $(A + 1 = 1)$

- **Commutative:** $(A + B = B + A)$

- **Associative:** $(A + (B + C) = (A + B) + C)$

- **Idempotent:** $(A + A = A)$

- **Distributive:**

- Over AND: $(A + (B \text{ AND } C) = (A + B) \text{ AND } (A + C))$

- Over OR: $(A + (B + C) = (A + B) + C)$

-- **Maxterm** = $(A^- + B^- + C^- + D^-)$

- **Product of sums:** $Y = (A^- + B^- + C^- + D^-) \cdot (A + B + C + D) \cdot (A^- + B^- + C^- + D^-)$

Priority Encoder

Definition:

A Priority Encoder is a combinational circuit that encodes the position of the highest-priority active input into a binary output.

Key Features:

- **Inputs:** (2^n) input lines; only one active at a time is expected.

- **Outputs:** (n) output lines representing the binary code of the highest-priority input.

Combinational Logic (cont)

- **Priority:** Higher-order inputs have precedence over lower-order inputs.

- **Enable Output:** Indicates if any input is active (optional).

Logic Expressions (4-to-2

Priority Encoder):

- $Y_1 = D_3 + D_2$

- $Y_0 = D_3 + \{D_2\}^- \cdot D_1$

- Enable Output $(E = D_3 + D_2 + D_1 + D_0)$

Applications:

1. **Interrupt Handling:** Assigns priority to multiple interrupt signals in processors.

2. **Data Compression:** Encodes multiple inputs into fewer bits.

3. **Memory Decoding:** Selects the highest-priority address or resource.

Advantages:

- Handles multiple inputs with priority logic.

- Compresses input size efficiently.

Limitations:

- Requires additional handling if multiple inputs have the same priority.

- Extra circuitry needed for "no active input" conditions.

Causes of Delay in Circuit State Change (Low to High)

Definition:

Combinational Logic (cont)

Delays occur when a circuit element transitions from 0 (low) to 1 (high) due to physical and electrical factors.

Key Causes of Delay

1. **Propagation Delay:**

- Time taken for a signal to propagate through a circuit element.

- Affects overall circuit speed.

2. **Gate Capacitance:**

- Time required to charge or discharge the transistor's gate capacitance.

3. **Load Capacitance:**

- Higher load capacitance slows the charging/discharging process.

4. **Resistance of Interconnects:**

- Higher resistance in wires increases (RC) delay.

5. **Signal Rise Time:**

- Time taken for the signal to rise from 10% to 90% of its final value.

6. **Threshold Voltage:**

- Higher voltage thresholds cause slower transitions.

7. **Noise and Signal Integrity:**

- Crosstalk, interference, or power fluctuations can distort signals.

8. **Power Supply Voltage:**

- Lower voltages reduce drive strength, increasing delay.

9. **Temperature Effects:**

Combinational Logic (cont)

- High temperatures slow down transistor switching.

10. **Manufacturing Variations:**

- Fabrication inconsistencies can result in slower components.

11. **Clock Synchronization:**

- Skew or jitter in clock signals causes timing delays in sequential circuits.

12. **Parasitic Elements:**

- Unintended resistance, capacitance, or inductance contributes to delay.

Summary:

Delays result from intrinsic (e.g., propagation, capacitance) and external factors (e.g., noise, temperature). Optimizing design and materials can mitigate these delays.

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Combinational Logic (cont)

- **Maxterm** = $(A + B + C + D)$
- **Product of sums:** $Y = (A + B + C + D) \cdot (A + B + C + D) \cdot (A + B + C + D)$

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Sequential Logic

State Table for A, B, Clock and XOR gate what is Q?

To determine the contents of the output register Q, we'll use a **state table** that describes the relationship between the inputs A, B, the clock, and the XNOR gate, along with the resulting Q.

XNOR Gate Logic

- The output of an XNOR gate is $A \oplus B$ (logical equivalence):

- $A \oplus B = 0: (A \oplus B)^- = 1$ (when $A = B$)

- $A \oplus B = 1: (A \oplus B)^- = 0$ (when $A \neq B$)

The output Q of the register will update on every clock edge based on the XNOR output.

State Table

Assume:

- Inputs A and B are parallel inputs (change at each clock cycle).

- Register Q is updated at every **positive clock edge** with the XNOR output.

