

Interpretation of Coefficients

Beta-0

- log(odds) when $X_1 = 0, \dots, X_i = 0$
- Often not interpretable (outside range of data)
- Sometimes can be thought of as background odds

Beta-1

- Difference in log(odds) for two groups differing in their level of $X-1$ by one unit, but otherwise similar for all other $X-i$ (same as the log of the OR comparing these two groups)
- Because of the properties of logarithms, beta-1 is also the log of the odds ratio for two groups differing in their level of $X-1$ by one unit, but otherwise similar for all other $X-i$
- $(\beta-1)(1)$ is the log odds ratio between two groups differing in $X-1$ by one unit, while $(\beta-1)(5)$ is the log odds ratio between two groups differing in $X-1$ by five units

$e^{\beta-1}$

Interpretation of Coefficients (cont)

- Odds ratio for two groups differing in their level of $X-1$ by one unit, but otherwise agreeing in their level of all other $X-i$
- Similarly: $(\beta-1)(5)$ is the log(odds) between two groups different in their value of $X-1$ by 5, and $e^{(\beta-1)(5)}$ is the odds ratio between two such groups
- $(e^{\beta-1})(1)$ is OR comparing one unit apart, while $(e^{\beta-1})^5$ is OR comparing five units apart

Interaction Terms

- Relationship between X and Y is moderated through Z
- This means the OR for Y between two groups that differ on X varies with Z

$$\log(\text{odds}(Y|X,Z)) = \beta_0 + \beta_X(X) + \beta_Z(Z) + \beta_{XZ}(X*Z)$$

Coefficient Interpretation

- Beta-0 still log(odds) of Y , given all $X-i$ are zero

Interaction Terms (cont)

- Beta-X is difference in log(odds) of Y between two groups differing by one unit of X , when $Z = 0$
- Beta-Z is difference in log(odds) of Y between two groups differing by one unit of Z , when $X = 0$
- Possible that X and/or $Z = 0$ outside of range of data, but still need to include this

Interaction Term

- BetaXZ is change in slope per 1 unit difference in X , comparing 1 unit differences in Z
- $e^{\beta(\text{interaction})}$ is ratio of the OR when interacting variable = 1 compared to when interacting variable = 0
- "The interaction term is the difference in log(OR) comparing situations where the interacting variable differs by one unit."
- Note that on the log scale, this is a difference, whereas on the OR scale, it is a ratio

Model Fitting

- Model coefficients estimated by achieving "minimum deviance"
- No general formula exists for this; software is needed to do this
- The beta-hats identified through this process are called the maximum likelihood estimates (MLEs) of the true beta-0 and beta-1

Likelihood Theory

- Provides tools for converting modeling assumptions into SE estimates
- Assumes that in population, Y and X really do have logistic relationship; however, can still get "best estimates" with minimum deviance (just no SEs/CIs)

Estimation Theory

- Use robust estimates to obtain SEs/CIs of coefficients in the model, even when true population is not a logistic relationship
- Point estimates will be same as model-based, but CIs are slightly different

