Trig Integrals

First, Look for potential **u-substitution** and potentially remove the du from original function. If the degree is an odd number, look at removing one term to manipulate the remaining term because it is a multiple of 2. Lowering the degree from 2 to 1 is possible using trig identities. Be observant of rewriting terms ex (1/___)

Improper Integrals

There are **three** types of Improper Integrals. **Type 1**: Integrate from 1 to Infinity/-Infinity. **Type 2**: Integrate from 0 to 1, typically improper at zero, or possibly identifying where denominator is zero. **Type 3**: is a combination of both. **Type 1** converges on the P-Series (1 / x^p) when P > 1. Diverges when P<= 1. **Type 2** converges when P < 1, diverges when P >= 1. **Now** when solving the integral look for u-substitution and changing the *x*-bounds to *u*-bounds. The *u* may be typically found in the denominator, but look for the *du* to replace. In the case that there is a *ln*, you will need to do Integration by Parts, then complete. When completing **Type 1** look to take the limit as *t->inf*, and replacing *inf* with *t* in the bounds. Be aware of Trig Identities as well. **Type 2** look for vertical asymptotes. Similarly, there will be u-sub needed. In the bounds split at the V.A and solve as it approaches the value from the left, and from the right. Remember changing the *x*-bounds to *u*-bounds if substituting.

Parametric - Arc Length

When given **x = f(t) and y = g(t)** use the formula, where alpha is less than t and beta is greater than t. First find the derivative of *x*, and the derivative of *y*, this is done how we have done in the past. Once we have both first derivatives, square them. This will allow us to plug into the formula and plug in our bounds. After doing so, look for *u*-substitution. If not easily accessible, take out the GCF inside the radical. Now we can split into two radical times each other. For example, rad($144t^2$+$144t^4$) = rad($144t^2$(1+t^2) = rad($144t^2$) times rad(1+t^2). Then look for *u*-substitution. Be aware of the changing bounds, from *t* to *u*. The compute anti-derivative. Also, look for $(A+B)^2$ formula to simpify.

Parametric - Area

Look at Formula. If **x = f(t) and y = g(t)**. Once finding the derivative of f(t), substitute into the formula, where bounds are in terms of *t*. Then, you may be able to distribute/simplify by multiplying (FOIL). Then compute the anti-derivative. Use similar techniques mentioned to compute anti-derivative.

Trig Substitution

First identify which identity to use. [Trig Identities]. Then find the value of *a^2* and *a*. Plug *a* into the formula selected, and then calculate the derivative. Solve the derivate for *dx*, and substitute *x* and *dx* back into the original function. Manipulate the radical to potentially rewrite using Trig Identities. **If** there is no obvious substitution, you will most likely need to complete the square. Once you have completed the square look to use the formula. Then follow the same steps as above. Be careful of the bounds in u-substitution.

Arc Length

Identify which Formula to Use. Referring to *x*-values but switch to *y*-values if about the *y* axis. Identify the bounds of the integrals, *x*-values we are integrating from. Then use f(x) and find the first derivative. After finding the first derivative, square it. This will allow you to substitute inside the radical in the formula. A simpler integral will allow *u*-sub inside the radical, and then compute the anti-derivative. Be aware of whether or not you are changing the bounds. In other situations manipulate the radical, remove the $[f^I(x)]^2$ term, and multiply it by its reciprocal to get 1, and then just 1 so it is equal to itself. For example, (1+$x^{-2/3}$) = $x^{-2/3}$($x^{2/3}$+1). Then due to properties of radicals, you can remove the part multiplying and so you have the radical($x^{-2/3}$) times radical(1+$x^{2/3}$). Then simplify and complete using *u*-substitution. Be aware of changing bounds and Trig Identities.

Parametric Sketch

Table Method, setup *t*, x(t), and y(t). Then solve for different values of *t* and sketch on a cardinal plane. **Obtain Regular x/y equation** Harder, and you solve for *t* in terms of the *x* equation and then substitute that into the y(t) formula. So you have *y* in terms of *x* not *t*.

Parametric - Tangent / (Horiz & Vert)

Look for the Formula. Setup using point slope form, y-y1 = m(x-x1) and needing a Point(x1,y1). First calculate the derivatives of x(t) and y(t). Then have the y(t) derivative as the num. and derivative of x(t) as the denom. This is the slope or *m*. **If you are given a t value**, you may solve for an *x1* point by substituting *t* into x(t). Likewise for y(t). Now you have a point on the curve and can plug into point-slope form. You can also substitute *t* into your slope formula, (dy/dx), to obtain the numerical slope, finishing your tangent equation. You can distribute and manipulate into y=mx+b form. **If you are given a point**, set the x1 value = x(t), and solve for all solutions of *t*. Do the same with y1 value given = y(t). The *t* value that is shared is the *t* used in the slope equation (dy/dx). Then obtain *m* and plug into point--slope form. **Horizontal** The numerator of the slop (*m*) = 0. Obtain the *t* value that is a zero, then using that *t* value substitute into x(t) and y(t) to get the P(x1,y1) the tangent occurs at. **Vertical** Same thing except find zeros of the denominator. **Finding Slope**, set the *m* equal to the desired slope. Then find *t*, plug into x(t) and y(t) for the point.

Partial Fraction Decomposition

First check the degree of the numerator and denominator. If the degree of the denominator is greater than the numerator you may begin Partial Fraction Decomposition. **If Otherwise** complete long division and begin P.R.D on the remainder. Factor the denominator so all solutions are present. Then setup P.F.D by doing separate integrals on the solutions and adding the integrals together. If the solution is liner (x+1), (x+2), $(x+2)^2$, then the numerator is just a constant letter, *A*. If the solution is non-liner, (x^2+1), then the numerator is *Cx+D*, where *C* and *D* are constants. Then complete the anti-derivative, and pay attention to **log rules**.

Surface Area - About x-axis

Look at X-axis Formula. **Alternate 1**. First identify what y = _. then R(x) = y, so R(x) is the function. **Now** that we have the function f(x), compute the first derivative. After doing so, square the first derivative. The answer will be substituted into the formula we are using. **After** substituting into the formula, look for *u*-substitution. Then compute the anti-derivative. If you have two sqrt function multiplying each other, then you can multiply the inside. Simplify after multiplying, and either preform the anti-derivative or look for *u*-sub. Pay attention to changing bounds if using *u*-sub. **If they gives x in terms of y LOOK HERE**. Here you will use the same function a little different. The radius is the distance between the curve and the axis-of-rotation. So here we use **Alternate 2** and R(y) = y. Then solving is the same from above just in terms of *y*. The bounds here are *y*-values.

Surface Area - About y-axis

Look at Y-Axis Formulas. **Alternate 1**. If given *y* in terms of *x* and the range in *x*-values, we want the radius in terms of *x*. So R = *x*. F(x) is the function given. Compute the first derivative of F(x), then square the result. This will allow us to substitute into our formula. Initially, look for *u*-sub inside the radical, if it is possible to cancel *du* on the outside, do so and compute the anti-derivative. Pay attention to changing the bounds when completing *u*-sub. **Alternate 2**. If given the range in terms of *y*, solve *x* in terms of *y*. R = *x* = F(y). The function is now in terms of *y*. Now compute the first derivative of F(y), and square the result. This can now be substituted into the formula. Initially, look for *u* substitution to replace inside the radical and cancel the function it is multiplied by. Pay attention to changing the bounds if doing *u*-sub.