Trig Integrals

Look to simplify the fraction. Remember different ways to write trig values, taking advantage of properties provided on the formula sheet. Also look to see if you can rewrite something inside the integral to something that has an easier anti derivative.

Improper Integrals

Three Types of Improper Integrals. Type 1: Integrate from 1 to infinity/-infinity. Type 2: integrate from 0 to 1. Type 3: Combination of both. Type 1 converges when the P (1/xp) is greater than one. (P > 1). Diverges for P <= 1. Type 2 converges when P < 1, and diverges for P >= 1. Look for changing bounds during u-substitution and separating integrals if there is a non-continuous point. For Type 1 take the limit as t->inf, and replace the inf bound with t.

Partial Fraction

Use constant over non-linear function, (x-1)2 is not non-linear, just multiple so include all multiples. Linear functions have a numerator of Bx+C. When using other technique, distribute and simplify all of the terms, then have x2 (A+B) if Ax2 and Bx2 are the only terms including x2. So A+B = coefficient of x2 in numerator provided. Look for shortcuts for taking anti-derivative of logs.

Trig Substitution

Use the formula to determine which type of Trig Sub we are doing, determining an a and using that to calculate x and dx. Then substitute everything back into the original integral. Now you have a Trig Integrals and refer back to them.

Integration By Parts

Formula Provided, use ILATE to choose U. Evaluate all x's if it is a definite integral with bounds. Look for Ln(x) and multiplication inside integrals to execute.

Power Series (Convergence)

First compute the ratio test by doing (an+1)/1 * 1/(an). Keep Change Flip. After computing remove the constants (terms without an n) and compute the limit. Distribute the remaining terms, and use this function to determine the radius. Set the funciton greater than -1 and less than 1. Then solve for x. The term not including the center is the radius. Manual check the end points for the interval of convergence.

Power Series (Representation)

Manipulate into the form (a/1-r). Then substitute into the Geometric Series and simplify. If given an x-value in the numerator, separate into two functions and complete expansion on one and then multiply by numerator. If given multiple of x in the denominaotr, factor out the coefficient but leave it in the r of the formula a/1-r. If the denominator simplifies, perform PFD and do expansion of both.

Arc Length

Identify which formula to use based on x or y integration. If you are given x values then you will most likely be doing x-integration with x bounds. Y values will be doing y-integration with y bounds. Use similar integral techniques referred to in other sections to solve.

Area Between Curves

Identify if you are doing x or y integration. If you are doing x integration then the bounds are in x and you take Vertical Slices (Upper – Lower) for your height. If you are doing y integration then the bounds are in y and you take Horizontal Slices (Rightmost – Leftmost) for your height. The larger (Upper/Rightmost) becomes f(x) and smaller (Lower/Leftmost) becomes g(x).

Volume: X-axis (Y=?)

Rotating around a horizontal axis look to do disk/washer method (vertical slices & dx). Where the radius is the function and if its washer do the ($Upper^2$ - $Lower^2$). Radius becomes (bounded - function) if it is above the x-axis.

Volume: Y-axis (X=?)

Vertical axis of rotation, use the cylinder shell method. The height is the function given, or if its bounded it is (bounded - function). The radius is the distance from the axis of rotation. So if it is Y a-axis, radius is x, if it is a different x-value, it becomes (bounded - x).

Surface Area - About x-axis

If you are given y in terms of x and in a range of x-values use Alternate X1. Here we can substitute into the formula and use bounds in terms of x. If you are given x in terms of y and a range of y-values use Alternate X2, substituting in the function g(y) and doing y-integration, with y-value bounds. Look for u-sub and multiplying two radicals to simplify. Pay attention to the bounds and simplification.

Surface Area - About y-axis

If you are given y in terms of x and in a range of x-values use Alternate Y1. Here we can substitute into the formula and follow the methods of Surface Area listed previously. If you are given x in terms of y and a range of y-values use Alternate Y2, substituting in the function g(y) and doing y-integration.

Work: Pumping liquid

Create a line next to tank having the upmost point being 0 and decreasing downward. The distance the water must travel is now x, and our bounds are where we need to travel from. If it is from the entire tank go from top to bottom, however if it is 1 less than full, go from 1 to the height. Use the equation of force and that Volume for each shape. The radius is half of the diameter. Plug in and take constants out and solve. If a cone, you may have to use similar triangles, look for formula.

Work: Lifting Chain

Lifting cable to top, Find the Force in weight of cable/length of cable. The force becomes that answer * x (distance). The bounds of the integral is the distance from the top to the bottom. Answer in ft/lbs. If only pulling half the rope to the top, take the integral from 0 to half using normal function, but then add the integral from half to the bottom, and substitute the half # for the distance (x) in the formula.

Work: Spring

The force required to maintain a spring is f(x) = kx. If given a force in N, find the distance from normal height to new length in meters. F(distance) = force in N., distance*K = force, solve for k. Substitute k back into f(x) = kx. Compute the integral from the desired distances of work done in meters. Final answer in J.

Parametric

Use the multiple equations for parametric. Bounds are in terms of t. Find tangent: Take derivative of x(t) and y(t) and have y'(t) as num and x'(t) as denom. That is our m (slope). If given a t value plug into x(t) and y(t) to get point cords. Substituting t into m will give you slope at a point. **Horizontal** is num = 0. **Vertical** is denom = 0. Make sure the point is not undefined, if it is take limit and do L'Hopitals.

Polar: Area

If they are asking for area of circle and not the other circle. One integral would be from the point of intersection to the next point of intersection, following the polar curve. And then subtracting the integral of circle not included from the point of intersection to the value of its next point of intersection. Ex) sin at zero is pi and cos at zero is pi/2. If they are asking for the area in between take the integral from the lower point of intersection and go to the point of intersection and add the integrals so you just have the smaller area. If it is a curve that loops itself, find the area of the top half and subtract when it loops back. Then multiply by 2.

Polar: Tangent Line

Given an r function, determine the angle where the points intersect (tangent occurs). First, take the derivative of the r function, r'(x). Then substitute the angle into our r function to determine the variable r. Using x=rcos(theta) and y=rsin(theta) substitute in the variable r and the angle. These will give the x,y cords for the y-y1=m(x-x1). Now solve for m. M=(r'(x)*sin(theta) + r*cos(theta)) / (r'(x)*cos(theta) - r*sin(theta)). Now plug in our terms and solve for m. Enter into formula.