

### ODE

#### Boundary Value Problem

values of dependent variable at more than one value of the independent variable

Turn it into an initial condition problem

#### Shooting Method

guess initial condition for the arbitrary variable

$$z = dT/dx$$

linear interpolation if you can

initial conditions satisfy boundary values

#### Finite Methods

approximate the derivative using one of the finite methods

reduce it to a system of linear equations

more computationally efficient than shooting method

shooting method: 1- solve RK4 multiple times 2- interpolate

### PDE

more than 1 independent variable

#### Elliptic Model

Laplace equation if equal to 0

Poisson's equation if not equal to 0

$$d^2T/dx^2 + d^2T/dy^2 = 0$$

not affected by time --x,y independent

#### Parabolic Model

$$dT/dt = K' (d^2T/dx^2)$$

time is a factor --x,t independent

#### Hyperbolic Model

$$d^2y/dx^2 = (1/c^2) (d^2y/dt^2)$$

waveform --x, t are independent

#### Elliptic Model

1 boundary values --> closed system or 2 secondary variable

maximum of 5 non-zeros per equation

Gauss sieedel: does not take into account zeros + DDS

### PDE (cont)

centered difference

Can i find one independent of other values?  
NO

without borders unknowns increase

centered difference equations (depends on order)

flux : derivative - insulated (=0)

### Interpolation

fit 1 function to all points

given points without function

increase accuracy, decrease step size or increase order

3 pts  $f(x) = ax^2 + bx + c$  - substitute points - system of linear equations (GE- GJ-inverse)

Alternative function representation

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2) + \dots$$

based on Taylor series

bs represent the slopes

efficient - quick

### Splines

fit a function to each interval

used for large datapoints--to avoid kinks

#### Linear Splines

$$f(x) = f(x_0) + m(x - x_0)$$

interval surrounds point

issues:

linearizing a non-linear function, oversimplifies behavior

discontinuity at the intermediate points - slope is no the same on either side

#### Quadratic Splines

minimum of 2 intervals or 3 points

$$f(x) = a_1x^2 + b_1x + c_1$$

3n unknowns -- n is # of intervals

(2n equations ) substitute points in formulas

### Splines (cont)

(n-1 equations) establish continuity with the slope at the intermediate points

assumption:  $a_1 = 0$

minimal effect on other intervals

under determined system by 1 equation

intermediate points are not independent

System of linear equations --do not use iterative methods (not DDS)

all functions are dependent

#### Cubic Splines

most popular method

minimum of 3 intervals or 4 points

4n equations - undetermined by 2 equations

assume 2nd derivative of outer points is 0

Alternative - Lagrange

$$(x_i - x_{i-1}) f''(x_{i-1}) + 2(x_{i+1} - x_i) f''(x_i) + (x_{i+1} - x_i) f''(x_{i+1}) = \frac{6}{(x_{i+1} - x_i)} [f(x_{i+1}) - f(x_i)] + \frac{6}{(x_i - x_{i-1})} [f(x_i) - f(x_{i-1})]$$

$$f(x) = \frac{f''(x_{i-1})}{6(x_i - x_{i-1})} (x_i - x)^3 + \frac{f''(x_i)}{6(x_i - x_{i-1})} (x - x_{i-1})^3 + \left[ \frac{f(x_{i-1})}{(x_i - x_{i-1})} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6} \right] (x_i - x) + \left[ \frac{f(x_i)}{(x_i - x_{i-1})} - \frac{f''(x_i)(x_i - x_{i-1})}{6} \right] (x - x_{i-1})$$

solve all second derivatives first

all related by continuity