ODE

Boundary Value Problem

values of dependent variable at more than one value of the independent variable

Turn it into an initial condition problem

Shooting Method

guess initial condition for the arbitrary variable

z= dT/dx

linear interpolation if you can

initial conditions satisfy boundary values

Finite Methods

approximate the derivative using one of the finite methods

reduce it to a system of linear equations

more computationally efficient than shooting method

shooting method: 1- solve RK4 multiple times 2- interpolate

Interpolation

fit 1 function to all points

given points without function

increase accuracy, decrease step size or increase order

3 pts f(x)=ax^2+bx+c - substitute points - system of linear equations (GE- GJ- inverse)

Alternative function representation

f(x)= b0 + b1(x- x0) +b2(x-x0)(x-x1)+ b3(x-x0)(x-x1)(x-x2) +...

based on Taylor series

bs represent the slopes

efficient - quick

PDE

more than 1 independent variable

Elliptic Model

Laplace equation if equal to 0

Poisson's equation if not equal to 0

$d^2T/dx^2 + d^2T/dy^2 = 0$

not affected by time --x,y independent

Parabolic Model

$dT/dt = K' (d^2T/dx^2)$

time is a factor --x,t independent

Hyperbolic Model

$d^2y/dx^2 = (1/c^2) (d^2y/ dt^2)$

waveform --x, t are independent

Elliptic Model

1 boundary values --> closed system or **2** secondary variable

maximum of 5 non-zeros per equation

Gauss siedel: does not take into account zeros + DDS

centered difference

Can i find one independent of other values? NO

without borders unknowns increase

centered difference equations (depends on order)

flux : derivative - insulated (=0)

Splines

fit a function to each interval

used for large datapoints--to avoid kinks

Linear Splines

f(x)= f(xo) + m(x-xo)

interval surrounds point

issues:

linearizing a non-linear function, oversimplifies behavior

discontinuity at the intermediate points - slope is no the same on either side

Quadratic Splines

Splines (cont)

minimum of 2 intervals or 3 points

f(x)= a1x^2 +b1x+c1

3n unknowns -- n is # of intervals

(2n equations) substitute points in formulas

(n-1 equations) establish continuity with the slope at the intermediate points

assumption: a1=0

minimal effect on other intervals

under determined system by 1 equation

intermediate points are not independent

System of linear equations --do not use iterative methods (not DDS)

all functions are dependent

Cubic Splines

most popular method

minimum of 3 intervals or 4 points

4n equations - undetermined by 2 equations

assume 2nd derivative of outer points is 0

Alternative - Lagrange

(xi - xi-1) f''(xi-1)+ 2(xi+1 -xi-1) f''(xi) +(xi+1 -xi) f''(xi+1) = (6/(xi+1 -xi)) [f(xi+1)-f(xi)] + (6/(xi - xi-1)) [f(xi-1) -f(xi)]

f(x)= (f''(xi-1)/6(xi -xi-1)) (xi -x)^3 + (f''(xi)/6(xi-xi-1)) (x- xi-1)^3 +[(f(xi-1)/(xi - xi-1)) - (f''(xi-1)(xi -xi-1)/6)] (xi - x)+ [(f(xi)/(xi -xi-1)) - (f''(xi)(xi- xi-1)/6)] (x-xi-1)

solve all second derivatives first

all related by continuity