## Systems of Linear Equations - Methods

| Elimination Methods | *Inverse Method* | Iterative Methods |
|---|---|---|
| Need scale system because system becomes more sensitive to round offs | solve multiple times for different constants | make unknowns the subject of equations |
| Maximum Coefficients on Main diagonal | Advantages | default all unknowns are 0 |
| *Gauss Elimination* | calculate inverse once | Dominant Diagonal System DDS |
| **1** forward elimination **2** back substitution | iterate for dynamic cases | DDS ensures convergence |
| eliminate what is below main diagonal | Limitations | *Gauss Seidel* |
| Issues | matrix has to have a solution | use updated values in equations |
| Zero at pivot - solution: switch rows | under-determined systems (# equations<#unknowns) | if system is converging |
| ill conditioned system - round off | do not have an inverse - infinite solutions | *Jacobi* |
| Limitations | Augmentation | update values at the end of each iteration |
| Lengthy- Cumbersome- Time consuming | [A:I] -> [I:A-] | help overcome divergence |
| 2 distinct steps | equations have to be linearly independent | Relaxation |
| *Gauss Jordan* | | Xinew= ~Xinew + (1-~)Xiold |
| eliminate what is above and below the main diagonal | | 0<~<2 |
| translate from coefficient matrix to identity matrix | | ~=0 diverging (initial conditions are most accurate) |

## Systems of Linear Equations - Methods (cont)

| | |
|---|---|
| Advantage: no need for back substitution | ~=1 regular |
| | ~=2 converging |
| | ~<1 diverging or converging with fluctuations |
| | ~>1 converging without fluctuations |
| | as system grows , ~ is close to 1 |

## Roots of Non linear Equations ---Numerical Methods

| Bracketed Methods | Open Methods |
|---|---|
| 2 initial guesses bracket the root | initial guesses do **not** have to bracket root |
| to check that intial guesses bracket root: f(xl)*f(xu)<0 | Newton Raphson |
| Bisection Method | Takes into account **1** initial guess **2** function behavior **3** rate of change |
| Xm= Xl+Xu / 2 | Xi+1= Xi- (f(xi)/ f'(xi)) |
| Limitations: | pitfalls |
| 1 miss roots | diverge due to infliction point |
| 2 inefficient (time consuming) | converge to local min/max |
| 3 if even # of roots between initial guesses are missed | jumping roots- converge to a different root |
| 4 disregard function behavior; function of initial guesses | if xi is close is zero, it will offshute |
| False Position | Limitation: differentiation |
| Xr= Xu - (f(xu) * (xl-xu))/(f(xl)-f(xu)) | Secant Method |
| in some cases, bisection may converge faster | xi+1 = xi - ((f(xi)* (xi-i -xi))/(f(xi-1)-f(xi)) |
| | Modified Secant |
| | 1 initial guess |
| | xi-1= xi + oxi |

| Roots of Non-linear Equations | |
|---|---|
| Analytical Solution | Graphical Solution |
| cannot solve complex equations | Visual Preceptions |
| Roots of an equation | Miss roots due to choice of window |
| find the value of independent variable when the dependent variable is zero. | |

| Systems of Linear Equations | |
|---|---|
| Graphical Solution | # equations = # unknowns |
| Visual perception - accuracy | 1 solution |
| Time consuming | # equations < # unknowns |
| impractical beyond 3D | infinite solutions |
| | # equations > # unknowns |
| | 1 solution (redundant equation) |
| | no solution - do not intersect |

| Systems of Linear Equations - Cranmer's Rule | |
|---|---|
| D = determinant of coefficients | Limitations |
| Dn = determinant of coefficients with n column replaced with B matrix | Time consuming |
| Singular System D=0 | if D=0 |
| 1 no solution | ill- conditioned system |
| 2 infinite solutions | D is close to 0 |
| | instruction is a region |
| | sensitive to round offs |