

Numerical Integration

Area under the curve

Single Integral

Trapezoidal Rule

fit linear function

Single Segment $A = (f(a) + f(b)) * (b - a) / 2$

Multiple Segments $A =$

$(f(x_0) + 2 \sum f(x_i) + f(x_n)) * (b - a) / 2n$

n is number of intervals- same width

increase accuracy.....increase intervals

Exception: -linear function -fluctuating function

as dx decreases, come closer to function

* inefficient but no limit on # of intervals

if non equal intervals, calculate separately and add

Simpson's 1/3 Rule

fit quadratic function

Single $A = (f(x_0) + 4f(x_1) + f(x_2)) * (b - a) / 6$

equidistant x1

Multiple $A =$

$(f(x_0) + 4 \sum_{\text{odd}} f(x_i) + 2 \sum_{\text{even}} f(x_i) + f(x_n)) * (b - a) / 3n$

of intervals is even

* even # of intervals

most popular bec accuracy is not that significant from 3/8 with less computation

Simpson's 3/8 Rule

fit cubic function

$A = (f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)) * (b - a) / 8$

1/3 rule is most widely used as computational efficiency it provides outweighs the accuracy provided by 3/8 rule

Trapezoidal rule can reach same accuracy of 3/8 rule by increasing number of intervals

* multiple of 3 # of intervals

Multiple Integral

Step 1 at $y=0$ find **A repeat**

Step 2 find A of A(y)

$T_{\text{avg}} = A(A(y)) / \text{area}$ or $T = A(A(y))$

Numerical Differentiation

Taylor series

$f(x_{i+1}) =$

$f(x_i) + f'(x_i)h + f''(x_i)h^2/2! + \dots + f^{(n)}(x_i)h^n/n!$

Exponential, you need infinite order because $f^{(n)}(x_i) = e^x$ which is never 0

First Forward difference

$f'(x_i) = (f(x_{i+1}) - f(x_i)) / h$

to increase accuracy, decrease h that will decrease the rest of Taylor series

First Backward difference

$f'(x_i) = (f(x_i) - f(x_{i-1})) / h$

First Centered difference

$f'(x_i) = (f(x_{i+1}) - f(x_{i-1})) / 2h$

Higher Order

First Forward difference

$f'(x_i) = -f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i) / (2h)$

First Backward difference

$f'(x_i) = 3f(x_i) - 4f(x_{i-1}) + f(x_{i-2}) / (2h)$

First Centered difference

$f'(x_i) = -f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2}) / (12h)$

Second Forward difference

$f''(x_i) = (f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)) / h^2$

Second Backward difference

$f''(x_i) = (f(x_i) - 2f(x_{i-1}) + f(x_{i-2})) / h^2$

Second Centered difference

$f''(x_i) = (f(x_{i+1}) - 2f(x_i) + f(x_{i-1})) / h^2$

* more accurate

Lagrange

fit interpolated polynomial then differentiate- function that passes by all points

general method

$f'(x) = pt_1 + pt_2 + pt_3$

$pt_1 = 2x - x_i - (x_i + 1) / ((x_i - 1) - x_i) ((x_i - 1) - (x_i + 1)) * f(x_{i-1})$

$pt_2 = 2x - (x_i - 1) - (x_i + 1) / (x_i - (x_i - 1)) (x_i - (x_i + 1)) * f(x_i)$

$pt_3 = 2x - (x_i - 1) - x_i / ((x_i + 1) - (x_i - 1)) ((x_i + 1) - x_i) * f(x_{i+1})$

Ordinary Differential Equations

Why solve numerically?

efficiency or cannot solve analytically

butterfly effect --sensitive to minor changes

chaotic systems --system sensitive to initial conditions but with predictable behavior

ODE with respect to 1 independent variable

PDE with respect to more than 1 independent variable

both can have many dependent variables

Euler's Method

$f(x_{i+1}) = f(x_i) + k_1 h$

to decrease error, either decrease timestep (h) or take more slopes

* to decrease timestep (h), take into account round off error propagates and computational efficiency

assume slope constant in interval

Heun's Method

second order

$y_{i+1} = y_i + k_{\text{avg}} h$

implicit method (y_{i+1} predictor) ---iterative

Midpoint Method

$y_{i+1} = y_i + k_2 h$

Explicit Method

$y_{\text{mid}} = y_i + (k_1 h) / 2$

Runge Kutte

generalized formula for methods

$y_{i+1} = y_i + \phi_i h$

ϕ_i is weighted average of slopes

number of ks reflects order

there are infinite methods

fourth order Runge Kutte

$y_{i+1} = y_i + h/6 (k_1 + 2k_2 + 2k_3 + k_4)$

Ralston's Method

$\phi_i = 1/3 k_1 + 2/3 k_2$

k_2 calculated at 3/4th of interval

third order Runge Kutte

$\phi_i = (k_1 + 4k_2 + k_3) / 6$

k_2 is midway and k_3 is at the end

systems of odes

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Ordinary Differential Equations (cont)

need to look at each independently but solve simultaneously

#initial conditions = # dependent variables

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