

### Parabola $f(x) = a(x-h)^2$

- $(-3-h)^2$   $h$  determines  $x$  axis
- $(2$  If  $a$  is positive parabola is up or to the right
- negatives= positive) up or to the right
- Vertex is at  $(h,k)$  if  $a$  is negative opens left or down
- $(x-3)^2 - 5 \rightarrow$  move to the right
- If parenthesis is addition, move to the left

### Graph the functions. Plot at least 3..

- Make a table for equation
- pick points for  $x$
- Solve
- graph answers
- should be in the form of  $f(x) = b^x$

### Graph the functions. plot at least.. (Rules)

- if  $b > 1$  its a exponential growth function
- if  $0 < b < 1$  its a exponential decay function
- if  $b > 1$  Domain =  $(-\infty, \infty)$
- if  $b > 1$  Range =  $(0, \infty)$
- if  $b > 1$  The line  $y=0$  is horizontal asymptote
- if  $b > 1$  Function passes through  $(0, 1)$

### Solve. $ax + bx + c = 0$

**2 Solving by Completing the Square**

Notice from Examples 3 and 4 that, if we write a quadratic equation so that one side is the square of a binomial, we can solve by using the square root property. To write the square of a binomial, we write perfect square trinomials. Recall that a perfect square trinomial is a trinomial that can be factored into two identical binomial factors.

<b>Perfect Square Trinomials</b>	<b>Factored Form</b>
$x^2 + 8x + 16$	$(x + 4)^2$
$x^2 - 6x + 9$	$(x - 3)^2$
$x^2 + 3x + \frac{9}{4}$	$(x + \frac{3}{2})^2$

Notice that for each perfect square trinomial in  $x$ , the constant term of the trinomial is the square of half the coefficient of the  $x$ -term. For example,

$x^2 + 8x + 16$	$x^2 - 6x + 9$
$\frac{1}{2}(8) = 4$ and $4^2 = 16$	$\frac{1}{2}(-6) = -3$ and $(-3)^2 = 9$

The process of writing a quadratic equation so that one side is a perfect square trinomial is called **completing the square**.

**EXAMPLE 5** Solve  $p^2 + 2p = 4$  by completing the square.

**Solution:** First, add the square of half the coefficient of  $p$  to both sides so that the resulting trinomial will be a perfect square trinomial. The coefficient of  $p$  is 2.

$$\frac{1}{2}(2) = 1 \text{ and } 1^2 = 1$$

Add 1 to both sides of the original equation.

$$p^2 + 2p + 1 = 4 + 1$$

$(p + 1)^2 = 5$  Add 1 to both sides. Factor the trinomial; simplify the right side.

We may now use the square root property and solve for  $p$ .

$$p + 1 = \pm \sqrt{5}$$

$p = -1 \pm \sqrt{5}$  Use the square root property. Subtract 1 from both sides.

Notice that there are two solutions:  $-1 + \sqrt{5}$  and  $-1 - \sqrt{5}$ .

### Shifting Parabolas

- $f(x) = ax^2 + bx + c$  is a parabola
- $f(x) = x^2 + k$
- $f(x) = (x-h)^2$
- $a > 0$  opens up is a vertical shift of  $f(x) = x^2$
- $h > 0$  shifted right vice versa
- $a < 0$  opens down  $k > 0$  shifted up vice versa

### Quadratic functions of the form

**Graphing the Parabola Defined by  $f(x) = ax^2$**

If  $a$  is positive, the parabola opens upward, and if  $a$  is negative, the parabola opens downward.

If  $|a| > 1$ , the graph of the parabola is narrower than the graph of  $y = x^2$ .

If  $|a| < 1$ , the graph of the parabola is wider than the graph of  $y = x^2$ .

### more parabolas

**3 Graphing  $f(x) = (x - h)^2 + k$**

As we will see in graphing functions of the form  $f(x) = (x - h)^2 + k$ , it is possible to combine vertical and horizontal shifts.

**Graphing the Parabola Defined by  $f(x) = (x - h)^2 + k$**

The parabola has the same shape as  $y = x^2$ . The vertex is  $(h, k)$ , and the axis of symmetry is the vertical line  $x = h$ .

**EXAMPLE 5** Graph:  $F(x) = (x - 3)^2 + 1$

**Solution:** The graph of  $F(x) = (x - 3)^2 + 1$  is the graph of  $y = x^2$  shifted 3 units to the right and 1 unit up. The vertex is then  $(3, 1)$ , and the axis of symmetry is  $x = 3$ . A few ordered pair solutions are plotted to aid in graphing.

$x$	$F(x) = (x - 3)^2 + 1$
1	5
2	2
4	2
5	5

### $x$ (possible extra terms here) = square root of $b$

**Square Root Property**

If  $b$  is a real number and if  $a^2 = b$ , then  $a = \pm \sqrt{b}$ .

**Helpful Hint**

The notation  $\pm 3$ , for example, is read as "plus or minus 3." It is a shorthand notation for the pair of numbers  $+3$  and  $-3$ .

**EXAMPLE 1** Use the square root property to solve  $x^2 = 50$ .

**Solution:**

$$x^2 = 50$$

$$x = \pm \sqrt{50} \text{ Use the square root property.}$$

$$x = \pm 5\sqrt{2} \text{ Simplify the radical.}$$

**Check:** Let  $x = 5\sqrt{2}$ .  $(5\sqrt{2})^2 \stackrel{?}{=} 50$   
 $25 \cdot 2 \stackrel{?}{=} 50$   
 $50 = 50$  True

Let  $x = -5\sqrt{2}$ .  $(-5\sqrt{2})^2 \stackrel{?}{=} 50$   
 $25 \cdot 2 \stackrel{?}{=} 50$   
 $50 = 50$  True

The solutions are  $5\sqrt{2}$  and  $-5\sqrt{2}$ , or the solution set is  $\{5\sqrt{2}, -5\sqrt{2}\}$ .

Steps: get it in the form of the equation by adding and subtracting different sides  
 Apply square root and the plus or minus sign

### Find the inverse

- Change  $f(x)$  to  $y$
- Switch  $x$  &  $y$
- Solve for  $y$
- Don't forget about cross multiplying

### Solve log equation.

- Convert to exponential form based off of note
- Simplify
- $2^3 = 8$  which is  $\log 8 = 3$

### Solve using Substitution.

- Substitute the same terms with a letter
- Solve for the letter ex:  $(x=2)$
- Replace the letter with what was in the equation
- Solve

### Solve the inequality using the test point method

The following steps may be used to solve a rational inequality with variables in the denominator.

**Solving a Rational Inequality**

- Solve for values that make all denominators 0.
- Solve the related equation.
- Separate the number line into regions with the solutions from Steps 1 and 2.
- For each region, choose a test point and determine whether its value satisfies the original inequality.
- The solution set includes the regions whose test point value is a solution. Check whether to include values from Step 2. Be sure *not* to include values that make any denominator 0.

**EXAMPLE 5** Solve:  $\frac{5}{x+1} < -2$

**Solution:** First we find values for  $x$  that make the denominator equal to 0.

$$x + 1 = 0$$

$$x = -1$$

Next we solve  $\frac{5}{x+1} = -2$ .

$$(x + 1) \cdot \frac{5}{x+1} = (x + 1) \cdot -2 \text{ Multiply both sides by the LCD, } x + 1.$$

$$5 = -2x - 2 \text{ Simplify.}$$

$$7 = -2x$$

$$\frac{7}{-2} = x$$

We use these two solutions to divide a number line into three regions and choose test points. Only a test point value from region  $B$  satisfies the original inequality. The solution set is  $(-\frac{7}{2}, -1)$ , and its graph is shown.

### Logarithms

log 1=1 Only solve multiple logs if they have the same base

log log(xy) = log(x) + log(y)  
 $b^x = x$

$b^{\log_{\text{sub } b} x} = x$

log 1=0

log b=1

power is what it's equal to  
 base is the sub

### f and g functions

replace f and g Horizontal line test  
 perform operation in middle

$(g \circ f)(x) = g(f(x))$  intersects more than once, not a function

### Graphing inverse

Look at > or < sign, determines what part of parabolas are the answer

Find the inverse

plot parabola from solved equation "y=..."

Make a table of points

### Solve the equation b = c

Get bases same by putting a power or square root or fraction

Cross out bases

Exponents become base

Solve equation

*Sometimes doesnt look like example, general it has 2 numbers raised to a power with an equal sign between*

### Quadratic Formula

**Quadratic Formula**  
 A quadratic equation written in the form  $ax^2 + bx + c = 0$  has the solutions  

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Solving a Polynomial Inequality

Think of it as a Quadratic equation (< OR > as a = sign)

Solve equation

Plot answers

Pick numbers from each A,B,C

if equation is tue part of solution

if false not part of solution

find and write out solution set

### Solving a Polynomial Inequality EX

**EXAMPLE 1** Solve:  $(x + 3)(x - 3) > 0$   
**Solution** First we solve the related equation,  $(x + 3)(x - 3) = 0$ .  

$$(x + 3)(x - 3) = 0$$
  

$$x + 3 = 0 \text{ or } x - 3 = 0$$
  

$$x = -3 \qquad x = 3$$
  
 The two numbers -3 and 3 separate the number line into three regions, A, B, and C.



Now we substitute the value of a test point from each region. If the test value satisfies the inequality, every value in the region containing the test value is a solution.

Region	Test Point Value	$(x + 3)(x - 3) > 0$	Result
A	-4	$(-1)(-7) > 0$	True
B	0	$(3)(-3) > 0$	False
C	4	$(7)(1) > 0$	True

The points in regions A and C satisfy the inequality. The numbers -3 and 3 are not included in the solution since the inequality symbol is >. The solution set is  $(-\infty, -3) \cup (3, \infty)$  and its graph is shown.



### Vertex Formula

First, isolate the x-variable terms by subtracting c from both sides.  

$$y = ax^2 + bx + c$$
  

$$y - c = ax^2 + bx$$
  
 Next, factor a from the terms  $ax^2 + bx$ .  

$$y - c = a\left(x^2 + \frac{b}{a}x\right)$$
  
 Next, add the square of half of  $\frac{b}{a}$ , or  $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ , to the right side inside the parentheses. Because of the factor a, what we really added was  $a\left(\frac{b^2}{4a^2}\right)$ , and this must be added to the left side.  

$$y - c + a\left(\frac{b^2}{4a^2}\right) = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right)$$
  

$$y - c + \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$$
 Simplify the left side and factor the right side.  

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$
 Add c to both sides and subtract  $\frac{b^2}{4a}$  from both sides.  
 Compare this form with  $f(x)$  or  $y = a(x - h)^2 + k$  and see that h is  $-\frac{b}{2a}$ , which means that the x-coordinate of the vertex of the graph of  $f(x) = ax^2 + bx + c$  is  $-\frac{b}{2a}$ .

**Vertex Formula**  
 The graph of  $f(x) = ax^2 + bx + c$ , when  $a \neq 0$ , is a parabola with vertex  

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

### Didn't go over

9.3-9.6

