LIMITS AND DERIVATIVES

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INTRO

We say lim(x→a⁻) f(x) is the expected value of f at x = a given the values of f near
x to the left of a. This value is called the left hand limit of f at a.
We say lim (x→a⁺) f(x) is the expected value of f at x = a given the values of
f near x to the right of a. This value is called the right hand limit of f(x) at a.
If the right and left hand limits coincide, we call that common value as the limit
of f(x) at x = a and denote it by lim(x→a) f(x).

LHL AND RHL

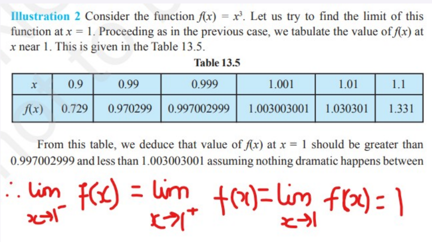
Illustration 2 Consider the function $f(x) = x^3$. Let us try to find the limit of this function at $x = 1$. Proceeding as in the previous case, we tabulate the value of $f(x)$ at x near 1. This is given in the Table 13.5.

Table 13.5

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.729	0.970299	0.997002999	1.003003001	1.030301	1.331

From this table, we deduce that value of $f(x)$ at $x = 1$ should be greater than 0.997002999 and less than 1.003003001 assuming nothing dramatic happens between

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = 1$$

1. A constant function takes the same value for all values of x, hence, limit will also be same
2. If value of lhl != rhl, limit is not defined
3. However, at a given point the value of a function and its limit may differ, even when both are defined

Algebra of limits

Limit of sum of two functions is sum of the limits of the functions	lim x→a [f(x) + g (x)] = lim x→a f(x) + lim x→a g(x)
Limit of difference of two functions is difference of the limits of the functions	lim x→a [f(x) – g(x)] = lim x→a f(x) – lim x→a g(x)
Limit of product of two functions is product of the limits of the functions	lim x→a [f(x) . g(x)] = lim x→a f(x). lim x→a g(x)
Limit of quotient of two functions is quotient of the limits of the functions (whenever the denominator is non zero)	lim x→a [f(x)/g(x)] = lim x→a f(x)/lim x→a g(x)

In particular as a special case of (iii), when g is a constant function such that g(x) = λ , for some real number λ , we have lim(x→a) [(λ.f)(x)]=λ. lim (x→a) f(x)

Limits of polynomial functions

A function f is said to be a polynomial function if f(x) is zero function or if f(x) = a0 + a1x + a2x2 +. . . + anxn, where aix are real numbers such that an ≠ 0 for some natural number n.
1. lim(x →a)x^n=a^n
2. let f(x)=a0+a1x+a2x^2...anx^n be a polynomial function.
then, f(x)=f(a)