

Definition

Leading Variables: variables corresponding to leading 1s.

Free Variables: variables with no pivot 1s.

Parameters: symbol (s, t, ...) for free variable.

Basic Solutions: vectors where s=1 and t,u,...=0.

Rank A: number of leading 1s.

If $A(m \times n)$ and $\text{rank } A = r$, there are $n-r$ free variables.

REF: leading 1s with 0s at the bottom.

RREF: leading 1s with 0s at the top and bottom.

Main Diagonal: $a_{11}, a_{22}, a_{33}, \dots$ entries of a matrix.

Diagonal Matrix: sq. matrix with entries outside main diagonal = 0.

Symmetric Matrix: sq. matrix where $A = A^T$.

Skew-symmetric: square matrix where $A^T = -A$.

Identity Matrix (I): square matrix where main diagonal entries = 1.

Zero Matrix: square matrix where all entries = 0.

Invertible Matrix: sq. matrix that has an inverse.

Elementary Row Operations

- I. Interchange two rows.
- II. Multiply a row by non-zero integer.
- III. Add multiple of a row with another row.

Transposition Properties

If $A = [a_{ij}]$, then $A^T = [a_{ji}]$

If A is $(m \times n)$, then A^T is $(n \times m)$.

$$(A^T)^T = A$$

$$(kA)^T = kA^T$$

$$(A+B)^T = A^T + B^T$$

Matrix Inverse Properties

$$AA^{-1} = I = A^{-1}A$$

$$\det A = ad - bc$$

$$\text{adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

A is non-invertible if $\det A = 0$.

$$A^{-1} = (1/\det A) \text{adj}A$$

$$Ax = b \Rightarrow x = A^{-1}b$$

$$[A \quad I] \rightarrow [I \quad A^{-1}]$$

Addition and Scalar Multiplication Properties

$$A+B = B+A \quad \text{commutativity}$$

$$A+(B+C) = (A+B)+C \quad \text{associativity}$$

$$0+A = A \quad \text{additive identity element}$$

$$A+(-A) = 0 \quad \text{additive inverse}$$

$$k(A+B) = kA+kB \quad \text{scalar distributivity}$$

$$(k+p)A = kA+pA \quad \text{scalar distributivity}$$

$$(kp)A = k(pA) \quad \text{scalar associativity}$$

$$1A = A \quad \text{multiplicative scalar identity}$$

Matrix-Vector Multiplication Properties

x is the solution to $Ax = b$

$$A(x+y) = Ax+Ay$$

$$A(ax) = a(Ax) = (aA)x$$

$$(A+B)x = Ax+Bx$$

Associated homogeneous system: $Ax = 0$

Matrix-Matrix Multiplication Properties

$$IA = A \text{ and } AI = A$$

$$A(BC) = (AB)C$$

$$A(B+C) = AB+AC$$

$$(B+C)A = BA+CA$$

$$a(AB) = (aA)B = A(aB)$$

$$(AB)^T = B^T A^T$$

$$\text{Idempotent: } A^2 = A$$

