Cheatography

# Discrete Math Cheat Sheet

by AviMaths (AviMathPerson) via cheatography.com/189338/cs/42865/

## Important relations

| | |
|---|---|
| Reflexive: | Shorthand: Ia ⊆ R Meaning: Every element is related to itself. for all a ∈ A, aRa holds ( R ⊆ {(a, a) \| a ∈ A} ) |
| Transitive: | Shorthand: ( R ∘ R = R2 ⊆ R ) Meaning: If ( (a, b) ∈ R ) and ( (b, c) ∈ R ), then ( (a, c) ∈ R ). (aRb and bRc) -> aRc |
| Symmetric: | Shorthand: ( R = $R^{-1}$ ) Meaning: If ( (a, b) ∈ R ), then ( (b, a) ∈ R ). When aRb <=> bRa |
| Antisymmetric: | Shorthand: ( R ∩ $R^{-1}$ ⊆ {(a, a) \| a ∈ A} ) Meaning: If ( (a, b) ∈ R ) and ( (b, a) ∈ R ), then ( a = b ). (aRb and bRa) -> (a = b) - This does not mean not-symmetric |

Equivalence relation is one where Reflexivity, Transitivity, and Symmetry all hold

## Cardinality

| Cardinality | Classification | Examples | | | | | |
|---|---|---|---|---|---|---|---|
| Aleph 0 | Countably infinite | $\mathbb{N}$ | $\mathbb{N}$ x $\mathbb{N}$ | $\mathbb{N}$ x $\mathbb{N}$ x $\mathbb{N}$ | $\mathbb{Z}$ | $\mathbb{Q}$ | $\mathbb{Q}$ x $\mathbb{Q}$ |
| Aleph | Uncountably infinite | (0,1) | {0,1}^$\mathbb{N}$ | P($\mathbb{N}$) | $\mathbb{R}$ | $\mathbb{R}$ x $\mathbb{R}$ | $\mathbb{C}$ |
| Finite | Countably finite | if A = {1}, \|A\| = 1 | {3,4,5} | {1,2,....1000} | | | |

## Combinatorics

| Case: | Order matters | Order doesn't matter |
|---|---|---|
| With repetition | n^k (case 1) | nCk (case 3) |
| Without repetition | nPk (case 2) | (k+n-1)C(k) (case 4) |

## Functions

Let f,g be two functions, (f:A -> B) , (g:B -> A)

| Function f | Horizontal line test | Classification | Invertibility | G is - inverse of f | Definition | English |
|---|---|---|---|---|---|---|
| Onto | Hits at least 1 point | Surjective | Right invertible | f ∘ g = Ib | {f(a) \| a ∈ A} = B every element in range (B) has a source | function that maps one or more elements of A to the same element of B |

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## Functions (cont)

| | | | | | | |
|---|---|---|---|---|---|---|
| One to One | Hits at most 1 point | Injective | Left Invertible | $g \circ f = Ia$ | if a1 != a2 then f(a1) != f(a2) or contrapositive if f(a1) = f(a2) then a1 = a2 | function that always maps the distinct element of its domain to the distinct element of its codomain |
| Onto and One to One | Hits exactly at 1 point | Bijective | Invertible | $g \circ f = Ia$, $f \circ g = Ib$ | $f^{-1} = g$ | function that is both injective and surjective |
| Identity Ia | Hits exactly at 1 point | Bijective | Invertible | $f \circ Ia = f = Ib \circ f$ | f(a) = a | function that always returns the value that was used as its argument, unchanged |

$(g \circ f)(a) = g(f(a))$ means g composed with f

## Set Theory

$A \cup B = \{x \in A \text{ or } x \in B \text{ or both}\}$
$A \cap B = \{x \in A \text{ and } x \in B\}$
$A \oplus B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$
$A - B = A \cap B^c = \{x \in A \text{ and } x \notin B\}$
Demorgan's laws:
$(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$
Associativity:
$A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$
Distributivity;
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

## Subsets

$A \subseteq A \cup B = B \cup A$
$B \subseteq A \cup B$
If $A \subseteq B$, then $A \cup B = B$
$A \cap B \subseteq A \subseteq A \cup B$
$B \cap A \subseteq B \subseteq A \cup B$

## Cartesian product:

$A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}$
an unordered set of sets of ordered pairs where a is in A, b is in B
if A = {1,2}, B = {2,3}, then A × B = {(1,2), (1,3), (2,2), (2,3)}
$A \times B \neq B \times A$ (unless A = B)
$A \cap (A \times B) = \emptyset$
$|A \times B| = |A| \times |B|$
Distribution:
$A \times (B \cup C) = (A \times B) \cup (A \times C)$
$A \times (B \cap C) = (A \times B) \cap (A \times C)$

## Order relation

Partial order If and only if all (Reflexivity, Transitivity, and Anti-symmetry) hold
clear hasse diagram can be drawn

## Order relation (cont)

items for which the relation doesn't hold will be drawn but not connected to the others in the diagram

Total/Linear order:
Partial order holds
Totality: For any ( $a, b \in A$ ), either ( $(a, b) \in R$ ) or ( $(b, a) \in R$ ).
In other words: For any two distinct elements a and b, either a is related to b ($a \leq b$), or b is related to a ($b \leq a$).
hasse diagram would be a straight line (all elements relate to one another in this set)

## Universal Set

Universal Set = U:
for any finite set A
$U = \{ x \in U \mid x \notin A \}$
$A^c = U - A$
$(A^c)^c = A$
$U^c = \emptyset$
$\emptyset^c = U$

## Relations

## Relations (cont)

R consists of all pairs in R but with their elements reversed. If (a,b) is in R, then (b,a) is in $R^{-1}$
Composition of relations:
$R \circ S = \{ (a, c) \mid \exists\, b : (a, b) \in R \text{ and } (b, c) \in S \}$
Set of pairs (a,c) such that exists an element b for which both (a,b) is in R and (b,c) is in S
$R \circ R = R2$ is a relation composed with itself
$(R \circ S) \circ T = R \circ (S \circ T)$ i.e it is associative (but not communitive)
$Ia \circ R = R$

## Order relation terms

ARB = (a,b) ∈ R
Identity: Ia = (a,a)
Ex: {(1,1),(2,2),(3,3), … }
Relation on set itself: R ⊆ A × A
ARA is another way to write it too.
Empty relation when R = ∅
implies that the relation R is empty, meaning it does not hold between any two pairs. It's essentially a relation with no elements.
Complete relation when R = A × B
implies that the relation R contains all possible pairs that can be formed by taking one element from set A and one element from set B. It's a relation where every element of A is related to every element of B.
Inverse relation is $R^{-1}$ = { (b, a) | (a, b) ∈ R }

| | | |
|---|---|---|
| Minimal | An element a is minimal if there is no b such that b precedes a. | Elements with nothing less than them (no predecessors) |
| Minimum | An element a is a minimum if for all b, a precedes b | Element that is less than everything else (either a set has 1 minimum or no minimum element) |
| Maximal | An element a is maximal if there is no b such that a precedes b | follows from minimal (with greater than) |
| Maximum | An element a is a maximum if for all b, b precedes a | follows from minimum (with greater than) |

## Power sets

The power set of A is denoted as P(A) or 2A
A ∈ P(A), ∅ ∈ P(A)
If |A| = n, then |P(A)| = 2n
|P(A)| = 2|A|
If A = ∅, then P(A) = {∅}

### Power sets (cont)

If A = {1,2}, then P(A) = {∅, {1}, {2}, {1, 2}}
|A| < |P(A)|

### Power set proofs

P(A) ∩ P(B) = P(A ∩ B)
Forward Inclusion: Let $X$ be an arbitrary element in $P(A) \cap P(B)$. By definition of intersection, $X$ belongs to both $P(A)$ and $P(B)$. This implies $X$ is a subset of both $A$ and $B$. Consequently, $X$ is also a subset of their intersection, $A \cap$ ☐☐. Thus, $X$ is an element of $P(A \cap$ ☐☐). Therefore, $P(A) \cap P(B) \subseteq P(A \cap B)$.
Reverse Inclusion: Let $Y$ be an arbitrary element in $P(A \cap B)$. By definition, ☐☐ is a subset of $A \cap B$, hence a subset of both $A$ and $B$. Consequently, $Y$ belongs to both $P(A)$ and $P(B)$. Thus, $Y$ is an element of $P(A) \cap$ ☐☐$(B)$. Therefore, $P(A \cap B) \subseteq P(A) \cap P(B)$.
Conclusion: Combining both directions of inclusion, we've demonstrated that $P(A) \cap P(B) \subseteq P(A \cap B)$ and $P(A \cap$ ☐☐$) \subseteq P(A) \cap P(B)$, implying $P(A) \cap P(B) = P(A \cap B)$. Thus, the equality holds.

P(A) ∪ P(B) ≠ P(A ∪ B)

Example of why these aren't equal:
A = {1}, B = {2}, A ∪ B = {1,2} => P(A ∪ B) = {∅, {1}, {2}, {1,2}}

P(A) = {∅, {1}}, P(B) = {∅, {2}} => P(A) ∪ P(B) = {∅, {1}, {2}}

### More combinatorics

number of ways to place k balls in n boxes.
P = permutation
C = combination
Order matters = sequence of choices

### More combinatorics (cont)

Order doesn't matter = if we picked ball 1 then ball 2… it would be equivalent to picking ball 2 then ball 1
With replacement = same item can be picked several times
Without replacement= each item is chosen at most, 1 time

**Case 1**
K times out of n objects
Number of functions from A to B
|A| = K, |B| = n
if A has 3 elements, and B has 5… we would get 5^3 total functions that can be defined

**Case 2**
K unique balls in n small boxes (can only fit 1 item in each box)
number of one to one functions from A to B

**Case 3**
K identical balls in n small boxes
Binomial coefficients

**Case 4**
Bars and stars
K identical balls in n numbered boxes (but each box can hold >= 0 balls)