Important relations

Reflexive:	Shorthand: Ia ⊆ R Meaning: Every element is related to itself. for all a ∊ A, aRa holds (R ⊆ {(a, a) | a ∊ A})
Transitive:	Shorthand: (R ∘ R = R2 ⊆ R) Meaning: If ((a, b) ∊ R) and ((b, c) ∊ R), then ((a, c) ∊ R). (aRb and bRc) -> aRc
Symmetric:	Shorthand: (R = R^{-1}) Meaning: If ((a, b) ∊ R), then ((b, a) ∊ R). When aRb <=> bRa
Antisymmetric:	Shorthand: (R ∩ R^{-1} ⊆ {(a, a) | a ∊ A}) Meaning: If ((a, b) ∊ R) and ((b, a) ∊ R), then (a = b). (aRb and bRa) -> (a = b) - This does not mean not-symmetric

Equivalence relation is one where Reflexivity, Transitivity, and Symmetry all hold

Cardinality

Cardinality	Classification	Examples					
Aleph 0	Countably infinite	ℕ	ℕ x ℕ	ℕ x ℕ x ℕ	ℤ	ℚ	ℚ x ℚ
Aleph	Uncountably infinite	(0,1)	{0,1}^ℕ	P(ℕ)	ℝ	ℝ x ℝ	ℂ
Finite	Countably finite	if A = {1}, |A| = 1	{3,4,5}	{1,2,....1000}			

Combinatorics

Case:	Order matters	Order doesn't matter
With repetition	n^k (case 1)	nCk (case 3)
Without repetition	nPk (case 2)	(k+n-1)C(k) (case 4)

Functions

Let f,g be two functions, (f:A -> B) , (g:B -> A)

Function f	Horizontal line test	Classification	Invertibility	G is - inverse of f	Definition	English
Onto	Hits at least 1 point	Surjective	Right invertible	f ∘ g = Ib	{f(a) | a ∊ A} = B every element in range (B) has a source	function that maps one or more elements of A to the same element of B

Functions (cont)

One to One	Hits at most 1 point	Injective	Left Invertible	g ∘ f = Ia	if a1 != a2 then f(a1) != f(a2) or contrapositive if f(a1) = f(a2) then a1 = a2	function that always maps the distinct element of its domain to the distinct element of its codomain
Onto and One to One	Hits exactly at 1 point	Bijective	Invertible	g ∘ f = Ia, f ∘ g = Ib	$f^{-1} = g$	function that is both injective and surjective
Identity Ia	Hits exactly at 1 point	Bijective	Invertible	f ∘ Ia = f = Ib ∘ f	f(a) = a	function that always returns the value that was used as its argument, unchanged

(g ∘ f)(a) = g(f(a)) means g composed with f

Set Theory

A ∪ B = {x ∈ A or x ∈ B or both}
A ∩ B = {x ∈ A and x ∈ B}
A ⊕ B = (A - B) ∪ (B - A) = (A ∪ B) - (A ∩ B)
A - B = A ∩ B^c = {x ∈ A and x ∉ B}
Demorgan's laws:
$(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$
Associativity:
A ∪ (B ∪ C) = (A ∪ B) ∪ C = A ∪ B ∪ C
Distributivity;
A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Subsets

A ⊆ A ∪ B = B ∪ A
B ⊆ A ∪ B
If A ⊆ B, then A ∪ B = B
A ∩ B ⊆ A ⊆ A ∪ B
B ∩ A ⊆ B ⊆ A ∪ B

Cartesian product:

A × B = { (a, b) | a ∈ A and b ∈ B }
an unordered set of sets of ordered pairs where a is in A, b is in B
if A = {1,2}, B = {2,3}, then A × B = {(1,2), (1,3), (2,2), (2,3)}
A × B ≠ B × A (unless A = B)
A ∩ (A × B) = ∅
|A × B| = |A| × |B|
Distribution:
A × (B ∪ C) = (A × B) ∪ (A × C)
A × (B ∩ C) = (A × B) ∩ (A × C)

Order relation

Partial order If and only if all (Reflexivity, Transitivity, and Anti-symmetry) hold
clear hasse diagram can be drawn

Order relation (cont)

items for which the relation doesn't hold will be drawn but not connected to the others in the diagram

Total/Linear order:
Partial order holds
Totality: For any (a, b ∈ A), either ((a, b) ∈ R) or ((b, a) ∈ R).
In other words: For any two distinct elements a and b, either a is related to b (a ≤ b), or b is related to a (b ≤ a).
hasse diagram would be a straight line (all elements relate to one another in this set)

Universal Set

Universal Set = U:
for any finite set A
U = { x ∈ U | x ∉ A }
A^c = U - A
$(A^c)^c$ = A
U^c = ∅
$∅^c$ = U

Relations

Relations (cont)

R consists of all pairs in R but with their elements reversed. If (a,b) is in R, then (b,a) is in R^{-1}
Composition of relations:
R ∘ S = { (a, c) | ∃ b : (a, b) ∈ R and (b, c) ∈ S }
Set of pairs (a,c) such that exists an element b for which both (a,b) is in R and (b,c) is in S
R ∘ R = R2 is a relation composed with itself
(R ∘ S) ∘ T = R ∘ (S ∘ T) i.e it is associative (but not communitive)
Ia ∘ R = R

Order relation terms

ARB = (a,b) ∈ R
Identity: Ia = (a,a)
Ex: {(1,1),(2,2),(3,3), … }
Relation on set itself: R ⊆ A × A
ARA is another way to write it too.
Empty relation when R = ∅
implies that the relation R is empty, meaning it does not hold between any two pairs. It's essentially a relation with no elements.
Complete relation when R = A × B
implies that the relation R contains all possible pairs that can be formed by taking one element from set A and one element from set B. It's a relation where every element of A is related to every element of B.
Inverse relation is R^{-1} = { (b, a) | (a, b) ∈ R }

Minimal	An element a is minimal if there is no b such that b precedes a.	Elements with nothing less than them (no predecessors)
Minimum	An element a is a minimum if for all b, a precedes b	Element that is less than everything else (either a set has 1 minimum or no minimum element)
Maximal	An element a is maximal if there is no b such that a precedes b	follows from minimal (with greater than)
Maximum	An element a is a maximum if for all b, b precedes a	follows from minimum (with greater than)

Power sets

The power set of A is denoted as P(A) or 2A
A ∈ P(A), ∅ ∈ P(A)
If |A| = n, then |P(A)| = 2n
|P(A)| = 2|A|
If A = ∅, then P(A) = {∅}

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Power sets (cont)

If A = {1,2}, then P(A) = {∅, {1}, {2}, {1, 2}}
|A| < |P(A)|

Power set proofs

P(A) ∩ P(B) = P(A ∩ B)
Forward Inclusion: Let X be an arbitrary element in $P(A) \cap P(B)$. By definition of intersection, X belongs to both $P(A)$ and $P(B)$. This implies X is a subset of both A and B. Consequently, X is also a subset of their intersection, A ∩ ☐ ☐. Thus, X is an element of $P(A$ ∩ ☐ ☐). Therefore, $P(A) \cap P(B) \subseteq P(A \cap B)$.
Reverse Inclusion: Let Y be an arbitrary element in $P(A \cap B)$. By definition, ☐ ☐ is a subset of $A \cap B$, hence a subset of both A and B. Consequently, Y belongs to both $P(A)$ and $P(B)$. Thus, Y is an element of $P(A)$ ∩ ☐ ☐(B). Therefore, $P(A \cap B) \subseteq P(A) \cap P(B)$.
Conclusion: Combining both directions of inclusion, we've demonstrated that $P(A) \cap P(B) \subseteq P(A \cap B)$ and $P(A$ ∩ ☐ ☐) ⊆ $P(A) \cap P(B)$, implying $P(A) \cap P(B) = P(A \cap B)$. Thus, the equality holds.

P(A) ∪ P(B) ≠ P(A ∪ B)

Example of why these aren't equal:
A = {1}, B = {2}, A ∪ B = {1,2} => P(A ∪ B) = {∅, {1}, {2}, {1,2}}

P(A) = {∅, {1}}, P(B) = {∅, {2}} => P(A) ∪ P(B) = {∅, {1}, {2}}

More combinatorics

number of ways to place k balls in n boxes.
P = permutation
C = combination
Order matters = sequence of choices

More combinatorics (cont)

Order doesn't matter = if we picked ball 1 then ball 2… it would be equivalent to picking ball 2 then ball 1
With replacement = same item can be picked several times
Without replacement= each item is chosen at most, 1 time

Case 1
K times out of n objects
Number of functions from A to B
|A| = K, |B| = n
if A has 3 elements, and B has 5… we would get 5^3 total functions that can be defined

Case 2
K unique balls in n small boxes (can only fit 1 item in each box)
number of one to one functions from A to B

Case 3
K identical balls in n small boxes
Binomial coefficients

Case 4
Bars and stars
K identical balls in n numbered boxes (but each box can hold >= 0 balls)