

### Ground Mereology Axioms

| axiom        | meaning   | defn.                            |
|--------------|---|----------------------------------|
| <b>M</b>     | Ground Mereology  |                                  |
| $Pxy$        | x is a part of y  |                                  |
| Reflexivity  | x is a part of itself   | $Pxx$                            |
| Antisymmetry | x and y can't be parts of each other, unless they are actually the same thing | $Pxy \wedge Pyx \rightarrow x=y$ |
| Transitivity | if x is a part of y, and y is a part of z, then x is a part of z              | $Pxy \wedge Pyz \rightarrow Pxz$ |

### Ground Mereology Definitions

| sym. | meaning         | defn.                               |
|------|-----------------|-------------------------------------|
| PP   | Proper Part     | $PPxy := Pxy \wedge \neg Pyx$       |
| O    | Overlap         | $Oxy := \exists z (Pzx \wedge Pzy)$ |
| U    | Underlap        | $Uxy := \exists z (Pxz \wedge Pzy)$ |
| OX   | Over-Crossing   | $OXxy := Oxy \wedge \neg Pxy$       |
| UX   | Under-Crossing  | $UXxy := Uxy \wedge \neg Pyx$       |
| PO   | Proper Overlap  | $POxy := OXxy \wedge OXyx$          |
| PU   | Proper Underlap | $PUxy := UXxy \wedge UXyx$          |

### Derived Statements

|                                |                       |
|--------------------------------|-----------------------|
| Overlapping is Reflexive       | $Oxx$                 |
| Overlapping is Transitive      | $Oxy \rightarrow Oyx$ |
| Proper Parts are not Reflexive | $\neg PPxx$           |

### Extensional Mereology

|   |  |  |
|---|--|--|
| <b>EM</b>   | Extensional Mereology  |  |
| Supplementation Axiom   | $\neg Pxy \rightarrow \exists z (Pzx \wedge \neg Ozy)$               |  |
| Weak Supplementation  | <b>EM</b> $\vdash PPxy \rightarrow \exists z (PPzy \wedge \neg Ozx)$ |  |
| If all the proper parts of X are proper parts of Y, X is part of Y        |  |  |
| If two objects have the exact same proper parts, they are the same object |  |  |

### Closed (Extensional) Mereology

|              |   |  |
|--------------|---|--|
| <b>CEM</b>   | Closed Extensional Mereology  |  |
| $\iota$      | description operator<br>$\iota x$ is "the unique x such that"   |  |
| $x+y$        | sum (or fusion)<br>$Oxy \rightarrow \exists x \forall w (Pwz \leftrightarrow (Pwx \wedge Pwy))$<br>defined as:<br>$\iota z \forall w (Owz \leftrightarrow (Owx \vee Owy))$                                    |  |
| $x \times y$ | product<br>$Uxy \rightarrow \exists z \forall w (Owz \leftrightarrow (Owx \vee Owy))$<br>defined as:<br>$\iota z \forall w (Pwz \leftrightarrow (Pwx \wedge Pwy))$  |  |
| $x-y$        | difference<br>$\exists z (Pzx \wedge \neg Ozy) \rightarrow \exists z \forall w (Pwz \leftrightarrow (Pwx \wedge \neg Owy))$<br>defined as:<br>$\iota z \forall w (Pwz \leftrightarrow (Pwx \wedge \neg Owy))$ |  |
| $U$          | universe<br>$\exists z \forall x (Pxz)$<br>defined as:<br>$\iota z \forall x (Pxz)$   |  |
| $\sim x$     | compliment<br>$U-x$   |  |

### General (Extensional) Mereology

|              |  |  |
|--------------|--|--|
| <b>GEM</b>   | General Extensional Mereology  |  |
| Fusion Axiom | $\exists x \phi \rightarrow \exists z \forall y (Oyz \leftrightarrow \exists x (\phi \wedge Oyx))$ |  |

### Ground Topology Axioms

|              |   |       |
|--------------|---|-------|
| <b>T</b>     | Ground Topology                                   |       |
| $Cxy$        | x is connect to y                                 |       |
| Reflexivity  | x is connected to itself                          | $Cxx$ |
| Symmetry     | $Cxy \rightarrow Cyx$                             |       |
| Transitivity | $Pxy \rightarrow \forall z (Czx \rightarrow Czy)$ |       |



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Published 26th October, 2016.

Last updated 24th October, 2016.

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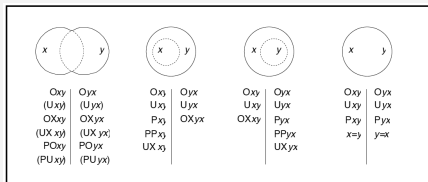
### Ground Topology Definitions

|     |                        |
|-----|------------------------|
| EC  | External Connection    |
| TP  | Tangential Part        |
| TPP | Tangential Proper Part |
| IP  | Internal Part          |
| IPP | Internal Proper Part   |
| E   | Enclosure              |
| IE  | Internal Enclosure     |
| TE  | Tangential Enclosure   |
| S   | Superposition          |
| PS  | Proper Superposition   |
| I   | Coincidence            |
| A   | Abutting               |

### Predicate Logic

|                   |               |
|-------------------|---------------|
| $\neg$            | not           |
| $\wedge$          | and           |
| $\vee$            | or            |
| $\forall$         | for every     |
| $\exists$         | there exists  |
| $\rightarrow$     | implies       |
| $:=$              | definition    |
| $\leftrightarrow$ | iff           |
| $\vdash$          | provable      |
| $\models$         | entails       |
| $\top$            | tautology     |
| $\perp$           | contradiction |

### Basic Patterns in Mereology



Credit: Varzi 1996, used without permission. The relations in parenthesis hold if there is a larger z including both x and y.

### Basic Patterns in Mereotopology

### Examples

|                      |   |
|----------------------|---|
| Part                 | Your finger is part of your hand  |
| Reflexivity          | Your finger is part of your finger  |
| Antisymmetry         | Your finger is part of your hand, but your hand is not part of your finger                                |
| Transitivity         | Your finger is part of your hand, and your hand is part of your body, so your finger is part of your body |
| Proper Part          | A tail is a proper part of a cat  |
| Overlapping          | Two roads overlap at their intersection   |
| Underlapping         | Your finger and thumb are underlapping parts of your hand   |
| Supplementation      | Road A is not part of Road B, because there is at least some of Road A that doesn't overlap Road B        |
| Weak Supplementation | Road A is not a proper part of Road B, because at least some of Road A is outside Road B                  |

### Alternate Notations

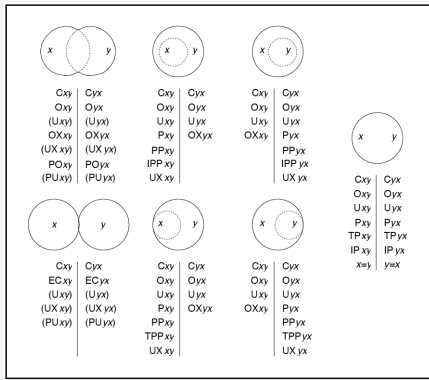
| symbol   | meaning                | from       |
|----------|------------------------|------------|
| $\ll$    | is a proper part of    | Simon 1987 |
| $<$      | is an improper part of | Simon 1987 |
| $\circ$  | overlaps               | Simon 1987 |
|          | is disjoint from       | Simon 1987 |
| $P_{xx}$ | is a part of           | Smith      |

### Mereological Operations

|                                   |                |             |
|-----------------------------------|----------------|-------------|
| $\cdot$                           | binary product | $x \cdot y$ |
| $+$                               | binary sum     | $x + y$     |
| $-$                               | difference     | $x - y$     |
| $\sigma x \ulcorner Fx \urcorner$ | fusion         |             |
| $\pi x \ulcorner Fx \urcorner$    | nucleus        |             |

### Smith (1996) Mereology Definitions

| sym. | meaning      | ex.     | defn.                            |
|------|--------------|---------|----------------------------------|
| P    | is a part of | $xPy$   |                                  |
| O    | overlaps     | $xOy$   | $\exists z(zPx \wedge zPy)$      |
| D    | discrete     | $xDy$   | $\neg xOy$                       |
| Pt() | is a point   | $Pt(x)$ | $\forall y(yPx \rightarrow y=x)$ |



Credit: Varzi 1996, used without permission. Seven basic patterns of the connection relationship.



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Published 26th October, 2016.

Last updated 24th October, 2016.

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