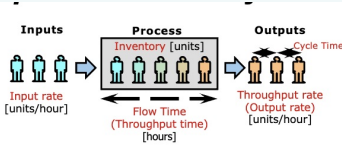


### Process Analysis



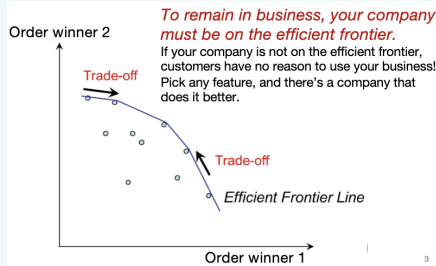
**Inventory** is the number of flow units contained in a process.  
**Flow time** is the time it takes for a unit to get through the entire process.  
**Throughput rate** is the rate at which the process is delivering output.  
**Cycle time** is the time between two consecutive units leaving the process.  
 • It is the inverse of throughput rate.  
**Capacity** is the maximum throughput rate possible  
**Bottleneck** is the resource with lowest capacity (highest processing time).  
**Process capacity** is the capacity of the bottleneck resource.

if demand rate is less than process capacity, throughput rate and cycle time are given by demand rate'

Flow time = add all times of the process  
 Throughput rate = inventory/time  
 Cycle time = time/inventory

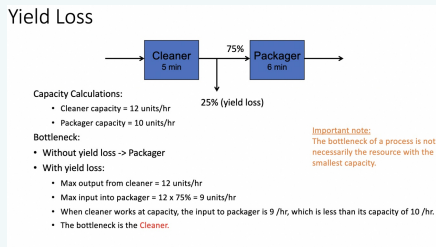
If there is an additional resource, the capacity of that part of the process doubles  
 Entire Process Time = Flow time + cycle time  $\times (x-1)$

### Efficiency Frontier + Order Winners



Order Winners: Quality, Speed, Flexibility, and Price

### Yield Loss



### Little's Law

$$\text{Inventory} = \text{Throughput Rate} \times \text{Flow Time}$$

$$\text{Days of Inventory} = (\text{Inventory} / \text{COGS}) \times 365$$

$$\text{Inventory Turnover} = \text{COGS} / \text{Inventory}$$

### Utilization

$$\text{Actual Utilization} = \frac{\text{Throughput rate (how much does resource produce)}}{\text{Capacity (how much can resource produce)}}$$

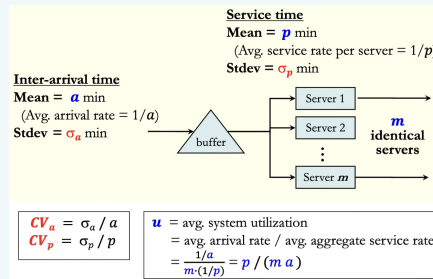
- Always < 100%
- Determines how much of its potential the resource is using

$$\text{Implied Utilization} = \frac{\text{Demand rate (request for the resource)}}{\text{Capacity (how much can resource produce)}}$$

- May be > 100%
- Determines the mismatch between demand and capacity

The bottleneck is defined as the resource with the highest implied utilization.

### VUT



### VUT

$$\text{Avg. Wait Time} = \text{Variability} \times \text{Utilization} \times \text{Service Time}$$

$$T_q \cong \left( \frac{CV_a^2 + CV_p^2}{2} \right) \times \left( \frac{u^{m+1}}{m(1-u)} \right) \times p$$

When  $m = 1$  (single server), VUT Equation reduces to:

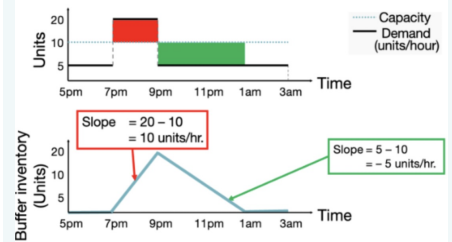
$$T_q \cong \left( \frac{CV_a^2 + CV_p^2}{2} \right) \times \left( \frac{u}{1-u} \right) \times p$$

### VUT Caveats

- VUT yields long-term, steady-state average waiting time.
- VUT applies only when  $u < 1$ . If  $u > 1$ , the system is unstable and we can't apply VUT! Use inventory build-up analysis instead (e.g., National Cranberry process)
- VUT assumes infinite buffer size
  - When buffer size is finite but large, VUT is a good approximation
  - When buffer size is small, use computer simulation to find wait time
- VUT equation is a good approximation, and it is an exact equation when  $m = 1$  and arrivals are "Poisson".

### Inventory Buildup

#### Inventory buildup diagram



Find total waiting time: area under the curve

### Normal Distribution

$$z = (x - \mu) / \sigma$$

$$x = z\sigma + \mu$$

$$z = \text{norm.s.inv}(\%)$$

### Newsvendor Model

#### Critical Fractile

- $C_o$  = Cost of over-stocking one unit (overage cost)
  - $C_u$  = Cost of under-stocking one unit (underage cost)
  - Our logic in the marginal analysis is:
    - Buy the  $Q + 1^{\text{st}}$  unit, if its Cost < Benefit:  $P(D \leq Q)C_o < P(D > Q)C_u$
    - Or equivalently:  $P(D \leq Q) < \frac{C_u}{C_u + C_o}$
    - Stop at  $Q^*$  when Cost  $\geq$  Benefit:  $P(D \leq Q^*) \geq \frac{C_u}{C_u + C_o}$
- The newsvendor logic:  
 -  $Q^*$  is the smallest quantity such that  $P(D \leq Q^*) \geq \frac{C_u}{C_u + C_o}$   
 Service level "Critical ratio"

### Newsvendor Model

#### Summary: Optimal Order Quantity

- With discrete demand
- With normal demand  $N(\mu, \sigma)$ 

Step 1: Find  $z$  with  $z = \text{Norm.s.inv}(C_u / (C_u + C_o))$   
 Step 2: Compute  $Q^* = \mu + z\sigma$

round up rule (round up optimal order quantity)

### Continuous Review Model

- Reorder point: Order  $Q^*$  once inventory hits ROP
  - Choose ROP so that service level (SL) is met
  - If  $D_L$  is normally distributed
    - Choose ROP so that  $P(D_L \leq ROP) = SL$
    - $SS = z\sigma_L$ ,  $ROP = dL + SS$ , where  $z = NORM.S.INV(SL)$
  - If  $D_L$  has discrete distribution
    - Choose ROP as smallest number where  $P(D_L \leq ROP) \geq SL$
    - $SS = ROP - E(D_L)$

Discrete Distribution: find cumulative probability, the quantity above the desired SL is your ROP

### Continuous Review Model

Continuous Review Model: Event Triggered Order

Service level is qualified as satisfying customer demand

Rule: if inventory = ROP order EOQ

EP = Lead Time

$ROP = d \text{ (units/day)} * L + z\sigma \text{ (L)}$

$\sigma L = \sigma d\sqrt{L}$

### Economic Order Quantity (EOQ)

D = Demand rate (unit/yr)

C = Cost of purchasing a unit (\$/unit)

S = Setup cost per order (\$)

H = Annual Holding Cost per unit of inventory (\$/unit \*year)

$H = iC$

i = Annual percentage holding cost

Q = Quantity of an order (units)

Number of Orders per year =  $D/Q$  ( / yr)

Annual Fixed (setup) cost =  $(D/Q) * S$  (\$/yr)

Average Inventory =  $Q/2$  (units)

Annual Holding cost =  $(Q/2) * H$  (\$/yr)

Annual purchasing cost =  $C * D$  (\$/yr)

$Q_{opt} = \sqrt{(2DS/H)}$

### Inventory Holding Strategies

- Inventory Pooling: centralizing inventory (keeping in one location)
- Delayed differentiation: keeping inventory of a base model and postpone final differentiation of products

### Periodic Review Model

Time Triggered Model: Order at specific time points

Exposure Period: time exposed to stock outs

Exposure Period =  $RP + LT$

Review Period (RP): amount of time it between each order

Target Stock Level =  $E[D]$  in EP + SS

Rule: @ time to order, order up to target stock level

$SS = z\sigma(d)$

$SL^* = C_u / (C_u + C_o)$

Amt to Order = Target Stock Level - Inventory

Periodic review may be necessary if: Too difficult/expensive to track current inventory (e.g. lack IT system) Supplier has bargaining power and/or capacity constraints → imposes order schedule Complex/rigid Shipping and Logistics Coordinating orders across multiple products from the same supplier

### Continuous Review vs Periodic Review

#### Continuous Review vs. Periodic Review

	Continuous Review	Periodic Review
Exposure Period	EP = L	EP = T + L
When to order	Inventory reaches ROP $ROP = dL + SS$	Fixed time: Every T days
How much to order	Fixed quantity: Order EOQ	$Q = \text{Target stock level} - \text{Net Inventory}$ $TSL = d(L + T) + SS$
Average Cycle Inventory	$Q/2 = EOQ/2$	$Q/2 = dT/2$
Safety Stock	$SS = z\sigma_{EP} = z\sigma_L$ [Normal] $SS = ROP - dL$ [Discrete]	$SS = z\sigma_{EP} = z\sigma_{L+T}$ [Normal] $SS = TSL - d(L + T)$ [Discrete]
Average Pipeline Inventory	dL	dL
Average Inventory (owned by firm)	<ul style="list-style-type: none"> <li>If pay upon ordering: <math>EOQ/2 + SS + dL</math></li> <li>If cash on delivery: <math>EOQ/2 + SS</math></li> </ul>	<ul style="list-style-type: none"> <li>If pay upon ordering: <math>dT/2 + SS + dL</math></li> <li>If cash on delivery: <math>dT/2 + SS</math></li> </ul>

