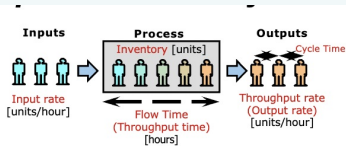## Process Analysis

Inventory is the number of flow units contained in a process.
Flow time is the time it takes for a unit to get through the entire process.
Throughput rate is the rate at which the process is delivering output.
Cycle time is the time between two consecutive units leaving the process.
- It is the inverse of throughput rate.
Capacity is the maximum throughput rate possible
Bottleneck is the resource with lowest capacity (highest processing time).
Process capacity is the capacity of the bottleneck resource.

if demand rate is less than process capacity, throughput rate and cycle time are given by demand rate'

Flow time = add all times of the process
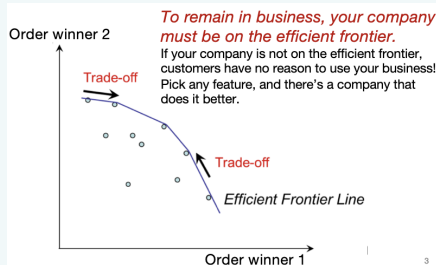Throughput rate = inventory/time
Cycle time = time/inventory
If there is an additional resource, the capacity of that part of the process doubles
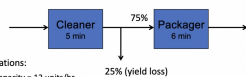Entire Process Time = Flow time + cycle time *(x-1)

Efficiency Frontier + Order Winners

Order Winners: Quality, Speed, Flexibility, and Price

Yield Loss

Little's Law

Inventory = Throughput Rate * Flow Time

Days of Inventory = (Inventory / COGS) * 365

Inventory Turnover = COGS/Inventory

Utilization

- Actual Utilization = $\dfrac{\text{Throughput rate (how much does resource produce)}}{\text{Capacity (how much can resource produce)}}$
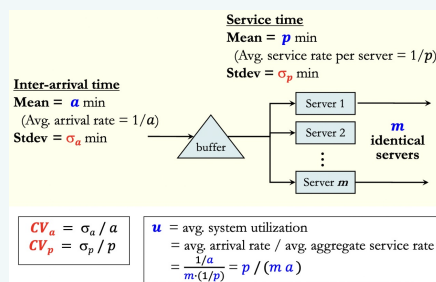
  - Always < 100%
  - Determines how much of its potential the resource is using

- Implied Utilization = $\dfrac{\text{Demand rate (request for the resource)}}{\text{Capacity (how much can resource produce)}}$

  - May be > 100%
  - Determines the mismatch between demand and capacity

The bottleneck is defined as the resource with the highest implied utilization.

VUT

$$CV_a = \sigma_a / a$$
$$CV_p = \sigma_p / p$$

u = avg. system utilization
= avg. arrival rate / avg. aggregate service rate
$= \dfrac{1/a}{m \cdot (1/p)} = p / (m\,a)$

VUT

Avg. Wait Time = **V**ariability × **U**tilization × Service **T**ime

$$T_q \cong \left(\frac{CV_a^{\,2} + CV_p^{\,2}}{2}\right) \times \left(\frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)}\right) \times p$$

| Variability effect | Utilization effect | Time scale |

When $m = 1$ (single server), VUT Equation reduces to:

$$T_q \cong \left(\frac{CV_a^{\,2} + CV_p^{\,2}}{2}\right) \times \left(\frac{u}{1-u}\right) \times p$$

VUT Caveats

- VUT yields **long-term, steady-state** average waiting time.

- VUT applies only when $u < 1$
  If $u > 1$, the system is unstable and we can't apply VUT!
  - Use inventory build-up analysis instead (e.g., National Cranberry process)

- VUT assumes **infinite buffer size**
  - When buffer size is finite but large, VUT is a good approximation
  - When buffer size is small, use computer simulation to find wait time

- VUT equation is a good **approximation**, and it is an exact equation when $m = 1$ and arrivals are "Poisson".

Inventory Buildup
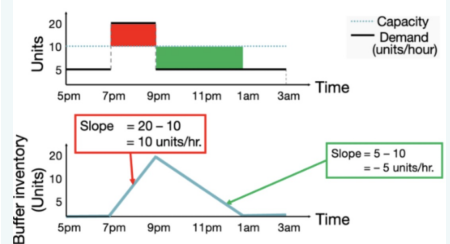
Find total waiting time: area under the curve

Normal Distribution

$z = (x-\mu)/\sigma$

$x = z\sigma + \mu$

z = norm.s.inv(%)

Newsvendor Model

Critical Fractile

C_o = Cost of over-stocking one unit (overage cost)
C_u = Cost of under-stocking one unit (underage cost)
- Our logic in the marginal analysis is:
  - Buy the $Q + 1^{\text{st}}$ unit, if its Cost < Benefit: $P(D \le Q)C_o < P(D > Q)\,C_u$
    Or equivalently: $P(D \le Q) < \dfrac{C_u}{C_u + C_o}$
  - Stop at Q^* when Cost ≥ Benefit $\qquad P(D \le Q^*) \ge \dfrac{C_u}{C_u + C_o}$
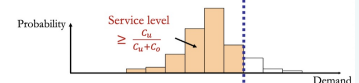 The newsvendor logic:
- Q^* is the smallest quantity such that $P(D \le Q^*) \ge \dfrac{C_u}{C_u + C_o}$
  (Service level) ("Critical ratio")

Newsvendor Model

Summary: Optimal Order Quantity

- With discrete demand
- With normal demand $N(\mu, \sigma)$
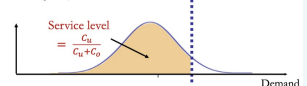
Step 1: Find z with $z = \text{Norm.s.inv}(C_u/(C_u + C_o))$
Step 2: Compute $Q^* = \mu + z\sigma$

round up rule (round up optimal order quantity)

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Continuous Review Model

- Reorder point: Order Q* once inventory hits ROP
  - Choose ROP so that service level (SL) is met
  - If D_L is normally distributed
    - Choose ROP so that $P(D_L \leq \text{ROP}) = \text{SL}$
    - $\text{SS} = z\sigma_L$, $\text{ROP} = dL + \text{SS}$, where $z = \text{NORM.S.INV(SL)}$
  - If D_L has discrete distribution
    - Choose ROP as smallest number where $P(D_L \leq \text{ROP}) \geq \text{SL}$
    - $\text{SS} = \text{ROP} - E(D_L)$

Discrete Distribution: find cumulative probability, the quantity above the desired SL is your ROP

Continuous Review Model

Continuous Review Model: Event Triggered Order

Service level is qualified as satisfying customer demand

Rule: if inventory = ROP order EOQ

EP = Lead Time

$\text{ROP} = d \text{ (units/day)} * L + z\sigma (L)$

$\sigma L = \sigma d\sqrt{L}$

Economic Order Quantity (EOQ)

D = Demand rate (unit/yr)

C = Cost of purchasing a unit ($/unit)

S = Setup cost per order ($)

H = Annual Holding Cost per unit of inventory ($/unit *year)

H = iC

i = Annual percentage holding cost

Q = Quantity of an order (units)

Number of Orders per year = D/Q (/ yr)

Annual Fixed (setup) cost = (D/Q) * S ($/yr))

Average Inventory = Q/2 (units)

Annual Holding cost = (Q/2) * H ($/yr)

Annual purchasing cost = C * D ($/yr)

$\text{Qopt} = \sqrt{(2DS/H)}$

Inventory Holding Strategies

1. Inventory Pooling: centralizing inventory (keeping in one location)

2. Delayed differentiation: keeping inventory of a base model and postpone final differentiation of products

Periodic Review Model

Time Triggered Model: Order at specific time points

Exposure Period: time exposed to stock outs

Exposure Period = RP + LT

Review Period (RP): amount of time it between each order

Target Stock Level = E[D] in EP + SS

Rule: @ time to order, order up to target stock level

$\text{SS} = z\sigma(d)$

$\text{SL*} = C_u / (C_u + C_o)$

Amt to Order = Target Stock Level - Inventory

Periodic review may be necessary if: Too difficult/expensive to track current inventory (e.g. lack IT system) Supplier has bargaining power and/or capacity constraints → imposes order schedule Complex/rigid Shipping and Logistics Coordinating orders across multiple products from the same supplier

Continuous Review vs Periodic Review

Continuous Review vs. Periodic Review

	Continuous Review	Periodic Review
Exposure Period	EP = L	EP = T + L
When to order	Inventory reaches ROP ROP = d L + SS	Fixed time: Every T days
How much to order	Fixed quantity: Order EOQ	Q = Target stock level - Net Inventory TSL = d (L + T) + SS
Average Cycle Inventory	Q / 2 = EOQ / 2	Q / 2 = d T / 2
Safety Stock	SS = z σ_{EP} = z σ_L [Normal] SS = ROP − d L [Discrete]	SS = z σ_{EP} = z σ_{T+L} [Normal] SS = TSL − d (L + T) [Discrete]
Average Pipeline Inventory	d L	d L
Average Inventory (owned by firm)	• If pay upon ordering: EOQ / 2 + SS + d L • If cash on delivery: EOQ / 2 + SS	• If pay upon ordering: d T / 2 + SS + d L • If cash on delivery: d T / 2 + SS