# linear algrebra Cheat Sheet

by afalita6 via cheatography.com/171839/cs/36095/

## Inverse of a matrix

| | |
|---|---|
| **Triangular or diagonal matrix** | 1/diagonal entries |
| **Permuted matrix** | P transpose |
| **Other** | rref ( [A eye()] ) |

## Multiplication of Matrix + angle

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| **Way 1** | A*B full multiplication |
| **Way 2** | [row A]*B |
| **Way 3** | [col A]*B |
| **Way 3** | B11*col(A1)+B21*col(A2) |
| **Find entry 2,3** | [row A2]*[columnB3] = 1 number |
| **Rank 1 matrix** | [a11*rowB1; a21*rowB1;a31*r-owB1] + ... |
| **Angle** | cos(theta) = (**v*****w**)/(\|\|**v**\|\|*\|\|**w**\|\|) |
| **Outer Product** | [column1]*[1 # #] find numbers that work |

## Linear Transformation and dependency

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| **Linear Indepe-ndent** | Linearly independent if rref(A) ----> #pivots = #row |
| **Linear transf-ormation (x and y given)** | T (u + v) = T (u) + T (v), T (cu) = cT (u), where c is a number. T is one-to-one if T(u)=0⇒u=0 T is onto if Col(T) = Rm. |

## Projections or Ax=b is inconsistent

| | |
|---|---|
| formula | A'*A*xhat=A'* |
| **Step 1** | rref ( [A'*A A'*b] ) |
| **Step 2** | xhat = last column of rref |
| **Step 3** | bhat = A*xhat --> bhat is the vector spaned A closest to v and the projection of the vector onto subspace |
| **Step 4** | be = b - bhat --> be is the vector perpendicular |
| **Step 4** | error vector/distance = norm (be) (1/sqr of components of b swuares) |

## Projections or Ax=b is inconsistent (cont)

| | |
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| **For regression** | step 1: f(x) = [x][b], step 2: A = [x.^0 ...] and y = given, step 3: do LSE and find xhat which will be a,b,c |

## Ax = b

| | |
|---|---|
| **Echelon form** | Leading entries in every row are farther to the right than the row above. To do = elimination steps |
| **Reduced Echelon form (rref)** | echelon + columns of leading entries are all 0 except the entry which must be a 1. To do = eliminations steps down to right, then left to top |
| **Ax=b with LU** | L = identity but a21 = -λ1, a31 = -λ2, a32 = -λ3. U = |

## Ax = b (A and b specified)

| | |
|---|---|
| **Echelon form** | Leading entries in every row are farther to the right than the row above. To do = elimination steps |
| **Reduced Echelon form (rref)** | echelon + columns of leading entries are all 0 except the entry which must be a 1. To do = eliminations steps down to right, then left to top |
| **Ax=b with LU** | L = identity but a21 = -λ1, a31 = -λ2, a32 = -λ3. U = echelon. Then do Ly=b - given (solve for y), then Ux=y (solve for x) |

## Ax = b (A and b specified) (cont)

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|---|---|
| **Ax=b with CR** | tMaybe not full rank. C = columns of A that have a pivot in R. R = rref form. To find x --> using R to find FV, pivots, and special solutions (if b not 0 do rref([A b])), if one soln is given then add that in gen sol and just do rref(A) |

## Eigenvectors and Eigenvalues

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|---|---|
| **v** | eigenvector |
| **λ** | eigenvalue |
| **Finding λ** | 1. Diag or triang = entries of diag. 2. 2x2 do λ = m +- sqrt (m^2 - p), where m = (a11+a-22)/2, and p = a11*a22 - a12*a21 |
| **Finding v** | rref ( [A - λ*eye ] ) and find FV, pivots, and ss |
| **Diagonali-zation** | A = P*D*P^(-1), where P = [eigenvectors] , D = diag(λ) |
| **When can we diagon-alize*** | Only when: square, real λ, and if repeated λ - look rref ( [A - λ*eye ] ) and only 1 pivot. |
| **A = Q*D*Q'** | Q = special solutions form rref ( [A - λ*eye ] ) for every λ, and then doing norm(q1) for all of them. D = diag(λs) |
| **Is λ an eigenvalue** | Do rref ( [A - λ*eye ] ) and has to be only 1 pivot (linearly dependent) |
| **Positive definite** | λs all positive |
| **Semipo-sitive definite** | λs all positive and at least a 0 |

## Eigenvectors and Eigenvalues (cont)

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| **Indefinite** | λ at least one is negative |

## Vector Spaces and Basis

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|---|---|
| **Subspace** | If u and v are in W , then u + v are in W , and cu is in W |
| **Basis B for V** | A linearly independent set such that Span (B) = V To show sthg is a basis, show it is linearly independent (rref(A) has NO FV) and spans(no row of 0's). |
| **Row(A)** | Space spanned by the rows of A: Row-reduce A and choose the rows that contain the pivots. Row(A) = R^n, dim = rank, Basis of Row = R in A = CR |
| **Col(A)** | Space spanned by columns of A: Row-reduce A and choose the columns of A that contain the pivots. Col(A) = R^m, dim = rank, Basis of Col = C in A = CR |
| **Null(A) / Vector in Null** | Solutions of Ax = 0. Row-reduce A. Null(A) = R^n, dim = n-rank, Basis of Null = rref(A), FV, pivots, special solutions |
| **LeftNull(A)** | Solutions of A'x = 0. Row-reduce A'. LeftNull(A) = R^m, dim = m-rank, Basis of LeftNull = rref(A'), FV, pivots, special solutions |
| **Rank(A)** | number of pivots |
| **Is v in Null** | do A*v and it needs to equal to vector 0 |
| **find v in ColA** | same vectors as in matrix |

## Vector Spaces and Basis (cont)

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|---|---|
| **Is v in col space of B** | is B*x=v consistent? do rref([B v]) and see if consistent |

## Gram-Schmidt steps

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|---|---|---|---|
| A | q1 = A(:,1) | Q = q1 | xhat = (q1'*-A(:,2))/(-q1'*q1) |
| ahat = Q*xhat | q2 = A(:,2) - ahat | Q(:,2) = q2 | Q(:,1) = 1/(q'1-*q1)*q1 |
| Q(:,2) = 1/(q'2-*q2)*q2 | Q = [ Q(:,1) Q(:,2) ] | R = Q'*A | if 3x3 keep going |

## Orthogonality

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| **v and u are othogonal** | if **v*u** = 0 |
| **W⊥:** | Set of v which are orthogonal to every w in W. |
| **Orthogonal projection:** | If {u1 · · · uk } is a basis for W , then orthogonal projection of y on W is: yˆ=(y·u1/u1*u1)+··-·+(y·u1/uk*uk), and y − yˆ is orthogonal to yˆ, shortest distance btw y and W is \|\|y−yˆ\|\| |
| **Basis of W⊥:** | basis of Null(Mw) |
| **Equalities between basis** | (RowA)' = NullA and vice versa. (ColA)'=LeftNullA and vice versa |