Relations

DEFINITION

A relation R, from a non-empty set A to another non-empty set B is mathematically as an subset of A × B. Equivalently, any subset of A × B is a relation from A to B. Thus, R is a relation from A to B Û R Í A × B Û R Í {(a, b) : a Î A, b Î B}

DOMAIN OF A RELATION

Let R be a relation from A to B. The domain of relation R is the set of all those elements a Î A such that (a, b) Î R " b Î B. Thus, Dom.(R) = {a Î A : (a, b) Î R " b Î B}. That is, the domain of R is the set of first components of all the ordered pairs which belong to R.

RANGE OF A RELATION

Let R be a relation from A to B. The range of relation R is the set of all those elements b Î B such that (a, b) Î R " a Î A. Thus, Range of R = {b Î B : (a, b) Î R " a Î A}. That is, the range of R is the set of second components of all the ordered pairs which belong to R.

CO-DOMAIN OF A RELATION

Let R be a relation from A to B. Then B is called the co-domain of the relation R. So we can observe that co-domain of a relation R from A into B is the set B as a whole.

REFLEXIVE RELATION

A relation R defined on a set A is said to be reflexive if a R a " a Î A i.e., (a, a) Î R " a Î A

SYMMETRIC RELATION

A relation R defined on a set A is symmetric if (a, b) Î R Þ (b, a) Î R " a, b Î A i.e., aRb Þ bRa (i.e., whenever aRb then bRa).

TRANSITIVE RELATION

A relation R on a set A is transitive if (a, b) Î R and (b, c) Î R Þ (a, c) Î R i.e., aRb and bRc Þ aRc.

Relations (cont)

EQUIVALENCE RELATION

Let A be a non-empty set, then a relation R on A is said to be an equivalence relation if (i) R is reflexive i.e., (a, a) Î R " a Î A i.e., aRa. (ii) For Let R is symmetric i.e., (a, b) Î R Þ (b, a) Î R " a, b Î A i.e., aRb Þ bRa. (iii)R is transitive i.e., (a, b) Î R and (b, c) Î R Þ (a, c) Î R " a, b, c Î A i.e., aRb and bRc Þ aRc

Functions

One-one function (Injective function or Injection)

A function f : A ® B is one-one function or injective function if distinct elements of A have distinct images in B. Thus, f : A ® B is one-one Û f(a) = f(b) Þ a = b, "a, b Î A Û a ≠ b Þ f(a) ≠ f(b) " a, b Î A.

Onto function (Surjective function or Surjection)

A function f : A ® B is onto function or a surjective function if every element of B is the f - image of some element of A. That implies f(A) = B or range of f is the co-domain of f. Thus, f : A ® B is onto Û f(A) = B i.e., range of f = co-domain of f.

One-one onto function (Bijective function or Bijection)

A function f : A ® B is said to be one-one onto or bijective if it is both one-one and onto i.e., if the distinct elements of A have distinct images in B and each element of B is the image of some element of A.

Matrices

PROPERTIES OF TRANSPOSE OF MATRICES :

(i)(A + B)T = AT + BT

(ii)(AT)T = A

(iii)(kA)T = kAT, where k is any constant

(iv) (AB)T = BT AT (v) (ABC)T = CT BT AT