

Relations

DEFINITION

A relation R , from a non-empty set A to another non-empty set B is mathematically as a subset of $A \times B$. Equivalently, any subset of $A \times B$ is a relation from A to B . Thus, R is a relation from A to $B \iff R \subseteq A \times B \iff R \subseteq \{(a, b) : a \in A, b \in B\}$

DOMAIN OF A RELATION

Let R be a relation from A to B . The domain of relation R is the set of all those elements $a \in A$ such that $(a, b) \in R \iff b \in B$. Thus, $\text{Dom.}(R) = \{a \in A : (a, b) \in R \iff b \in B\}$. That is, the domain of R is the set of first components of all the ordered pairs which belong to R .

RANGE OF A RELATION

Let R be a relation from A to B . The range of relation R is the set of all those elements $b \in B$ such that $(a, b) \in R \iff a \in A$. Thus, $\text{Range of } R = \{b \in B : (a, b) \in R \iff a \in A\}$. That is, the range of R is the set of second components of all the ordered pairs which belong to R .

CO-DOMAIN OF A RELATION

Let R be a relation from A to B . Then B is called the co-domain of the relation R . So we can observe that co-domain of a relation R from A into B is the set B as a whole.

REFLEXIVE RELATION

A relation R defined on a set A is said to be reflexive if $a R a \iff a \in A$ i.e., $(a, a) \in R \iff a \in A$

SYMMETRIC RELATION

A relation R defined on a set A is symmetric if $(a, b) \in R \implies (b, a) \in R \iff a, b \in A$ i.e., $a R b \implies b R a$ (i.e., whenever $a R b$ then $b R a$).

TRANSITIVE RELATION

A relation R on a set A is transitive if $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R$ i.e., $a R b$ and $b R c \implies a R c$.

Relations (cont)

EQUIVALENCE RELATION

Let A be a non-empty set, then a relation R on A is said to be an equivalence relation if (i) R is reflexive i.e., $(a, a) \in R \iff a \in A$ i.e., $a R a$. (ii) For Let R is symmetric i.e., $(a, b) \in R \implies (b, a) \in R \iff a, b \in A$ i.e., $a R b \implies b R a$. (iii) R is transitive i.e., $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R \iff a, b, c \in A$ i.e., $a R b$ and $b R c \implies a R c$

Functions

One-one function (Injective function or Injection)

A function $f : A \rightarrow B$ is one-one function or injective function if distinct elements of A have distinct images in B . Thus, $f : A \rightarrow B$ is one-one $\iff f(a) = f(b) \implies a = b$, " $a, b \in A \implies a \neq b \implies f(a) \neq f(b)$ " $a, b \in A$.

Onto function (Surjective function or Surjection)

A function $f : A \rightarrow B$ is onto function or a surjective function if every element of B is the f -image of some element of A . That implies $f(A) = B$ or range of f is the co-domain of f . Thus, $f : A \rightarrow B$ is onto $\iff f(A) = B$ i.e., range of $f =$ co-domain of f .

One-one onto function (Bijective function or Bijection)

A function $f : A \rightarrow B$ is said to be one-one onto or bijective if it is both one-one and onto i.e., if the distinct elements of A have distinct images in B and each element of B is the image of some element of A .

Matrices

PROPERTIES OF TRANSPOSE OF MATRICES :

$$(i) (A + B)^T = A^T + B^T$$

$$(ii) (A^T)^T = A$$

$$(iii) (kA)^T = kA^T, \text{ where } k \text{ is any constant}$$

$$(iv) (AB)^T = B^T A^T \quad (v) (ABC)^T = C^T B^T A^T$$

